Guide to answers for written examination in TSBB09 Image Sensors, 2020-01-16

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PART I: STANDARD CAMERAS & IR SENSORS

Exercise 1
A photon of wavelength λ_1 has higher energy than a photon of wavelength λ_2 when $\lambda_1 < \lambda_2$. This is because the energy E is inversely proportional to the wavelength λ: $E = h \cdot c/\lambda$, where h is Planck's constant and c is the speed of light. This means that there must be fewer detected photons from the first light source than from the second source to give the same total energy; $n_2 > n_1$.

Exercise 2
Pinhole camera.
Advantage: All 3D points projected onto the image are sharp.
Disadvantage: all light must pass through one small point, so only little light enters the camera which requires relatively long exposure time.
Lens based camera.
Advantage: light enters through a larger opening, allows shorter exposure time.
Disadvantage: only 3D points in, or close to, the object plane (defined by the geometry of the lenses) are projected sharply in the image.

Exercise 3
Vignetting is the "darkening" of an image towards its edges and corners.

Exercise 4
Global shutter.
The exposure and read-out happen frame after frame.
Rolling shutter.
The exposure and read-out happen line after line. Each successive line is exposed at successive points in time.
If the camera or the scene is moving, rolling shutter may distort the image.

Exercise 5
Photons hitting the sensor is a probabilistic phenomena. This results in Poisson distributed noise.

Exercise 6
Midwave and longwave infrared corresponds to thermal infrared.

Exercise 7
The fill factor is the percentage of the total sensor area which is light sensitive.
In practice, the light sensitive area of an image sensor cannot fill the entire detector area. Electronic components and wiring reduce the light sensitive area.
To enhance the fill factor, an array of micro-lenses can be placed front of the sensor array.

Exercise 8

- Thermal radiation that is reflected by the object.
- Thermal radiation emitted by the medium between the object and the camera (e.g. the air).
- Thermal radiation emitted by the camera itself.

PART II: GEOMETRY AND MULTIPLE VIEWS

Exercise 9
Flat objects.

Exercise 10
$\alpha/\beta = 1.1$ (and $\gamma = 0$ if it is a perfect rectangle).

Exercise 11
1 calibration plane determines 1 homography i.e. 8 parameters. There are 6 degrees of freedom in $[\mathbf{R}\,\mathbf{t}]$, 3 rotation angles and 3 translation directions. Consequently 8-6=2 equations are obtained for solving $\mathbf{K}$ from one calibration plane. Since there are 5 unknowns in $\mathbf{K}$, at least 3 calibration planes are needed.

Exercise 12
$\mathbf{C}$ is a 3×4-matrix that can be determined up to a scale factor, i.e. 11 unknowns in $\mathbf{C}$ need to be determined. One corresponding point-pair on the 3D calibration object and the image, contributes with 2 equations to solve for $\mathbf{C}$. Consequently, the minimum number of 3D points on the calibration object is 6.

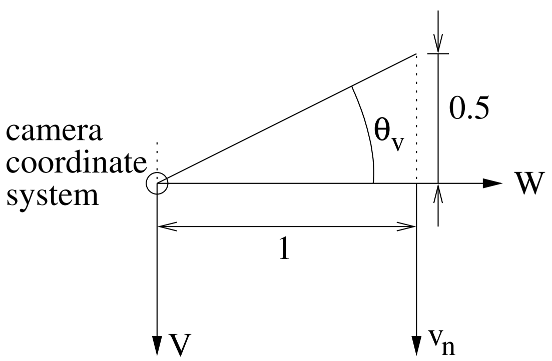
Exercise 13
In the panorama lab, we marked corresponding points in two images. The images were obtained at different rotation angles of the camera. The points in both images were then projected onto the unit sphere. The two sets of projected points were used as *indata*. The *outdata* was a matrix $\mathbf{R}$ that specified the rotation between the two images.

Exercise 14
Yes! Suppose that you are going to make a spherical panorama. From the images projected on the sphere, it is impossible to trace their respective $\mathbf{K}$ matrix. Nevertheless, it is necessary to have the same camera center of the two images.

Exercise 15
The normalized image coordinate (u_n, v_n) is transformered to the real image coordinate $(u, v) = (275, 175)$ according to

$$\begin{pmatrix} 275 \\ 175 \\ 1 \end{pmatrix} = \begin{pmatrix} 500 & 0 & 500 \\ 0 & 450 & 400 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} u_n \\ v_n \\ 1 \end{pmatrix}.$$

Solution to this equation system gives $(u_n, v_n) = (-0.45, -0.5)$. Therefore,
$\theta_u = \arctan(-0.45/1) = \arctan(-0.45)$ and
$\theta_v = \arctan(-0.5/1) = \arctan(-0.5)$, see figure.



Exercise 16

a) Corresponding points in *a) pattern* and *b) calibration image* should be chosen. Order is important, but not rotation. One solution is to let $(X_1, Y_1) = (0, 0)$, $(X_2, Y_2) = (5, 0)$, $(X_3, Y_3) = (10, 0)$ correspond to $(u_1, v_1) = (86, 26)$, $(u_2, v_2) = (165, 37)$, $(u_3, v_3) = (243, 48)$.

b)
- Determine corresponding points (X_i, Y_i), $i = 1, ..., N$ and (u_i, v_i), $i = 1, ..., N$.
- Denote equation (3) as $\mathbf{D} \cdot \mathbf{h} = \mathbf{f}$ and solve $\mathbf{h}$ by $\mathbf{h} = \mathbf{D}^+ \cdot \mathbf{f}$.
- Reshape $\mathbf{h}$ to $\mathbf{H}$.
- Measure the object endpoints (u_a, v_a) and (u_b, v_b) in pixels in *c) object image*.
- Compute corresponding real world coordinates (X_a, Y_a) and (X_b, Y_b) in centimeters by

$$s_a(X_a, Y_a, 1)^T = \mathbf{H}^{-1}\,(u_a, v_a, 1)^T,$$
$$s_b(X_b, Y_b, 1)^T = \mathbf{H}^{-1}\,(u_b, v_b, 1)^T.$$

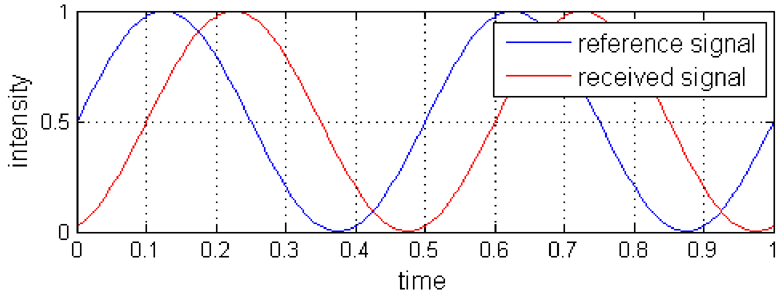
- The length of the object is finally given by $\sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2}$.

PART III: NON-STANDARD IMAGE SENSORS

Exercise 17
A set of pin-hole cameras arranged in a 2D array (matrix).

Exercise 18
Kinect uses an IR-light dot pattern which is different in every local neighborhood. It is designed to have as low autocorrelation as possible for shifts larger than the point size and in the interval of disparities that the system needs to deal with. The dot pattern in the exercise, on the other hand, repeats itself. There is a risk of choosing the wrong position for subsequent triangulation and range calculation.

Exercise 19
See the figure gelow. The phase difference between the reference signal and the received signal gives the time difference, which gives the range. There is an ambiguity in phase/time difference. In the figure, time difference can be 0.1 or 0.6. In case of three modulation frequencies (even two), this ambiguity can be avoided.



Exercise 20
1) Guess correspondences using current best guess of registration.
1b) Calculate the means of the two point sets, $\mathbf{X}_\mu$ and $\mathbf{Y}_\mu$. Subtract the means from the two point sets. This way, translation is removed.
2) Estimate the *rotation* $\mathbf{R}$ with the solution to the
'Orthogonal Procrustes Problem' (OPP).
2b) Update the translation as $\mathbf{t} = \mathbf{X}_\mu - \mathbf{R}\mathbf{Y}_\mu$.
3) Update correspondences, and goto 2.

Exercise 21
If $\mathbf{R} \cdot \mathbf{V}$ is negative, the ray is reflected away from the viewer. The intensity contribution $I_{specular}$ is then set to zero.

Exercise 22
I_0 is the light entering from the background (behind the volume).
I is light leaving the volume entering the camera or eye.
L is the line of integration along the light path.
$\mu(x, y)$ is the absorbtion function of the volume.
The equation models how light is attenuated while traveling through an absorbing material.

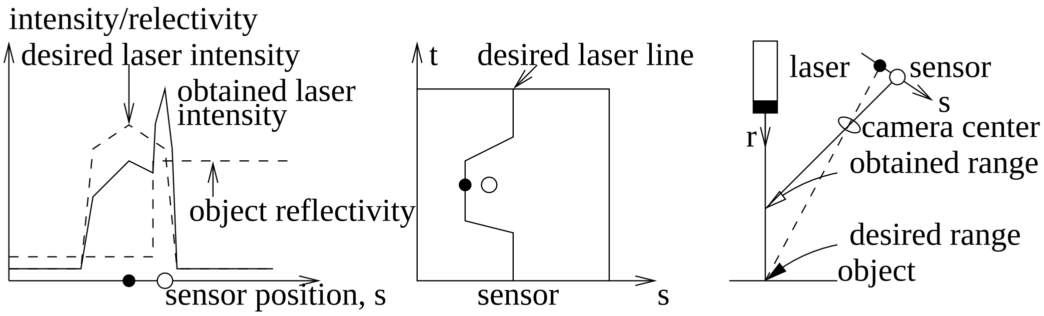
Exercise 23

A coded aperture replaces the (approximately) circular camera aperture with a more complex aperture pattern. In practice this is achieved by putting a mask onto the camera lens where the mask contains the aperture pattern. When taking a picture, defocused objects will then be convolved with the Airy disc and a scaled version f_k of the aperture pattern. Reconstructing a sharp image x from the unsharp image y is done by trying out various values for x and scales k (at different local positions) and minimizing the error $|f_k * x - y|^2$. Consequently:

- f_k is a scaled version of the aperture pattern, blurred with the Airy disc.
- k corresponds to the actual depth.
- f_k changes size with k.

Exercise 24

Due to the varying object reflectivity, the obtained laser intensity varies approximately as indicated in figure a). The laser intensity maximum gives the real position of the detected laser point and it is indicated with a white dot in a), and correspondingly in b) and c). Draw a line through the sensor position and the camera center. The cross-section of this line with the laser sheet is the range. Both the desired and the obtained position of the range are indicated in the figure.



a) b) sensor image c) equipment setup