Guide to answers for written examination in TSBB09 Image Sensors, 2019-01-16

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PART I: STANDARD CAMERAS & IR SENSORS

Exercise 1
$\alpha + \rho + \tau = 1$
ImageFormation_2018, slide 25

Exercise 2
The funtion is $\cos^4(\alpha)$, where α is the angle between the optical axis and a linje from the object to the optical center.
ImageFormation_2018, slide 75-79

Exercise 3
The fill factor is the percentage of the total sensor area which is light sensitive. To enhance the fill factor, an array of micro-lenses can be placed front of the sensor array.
ImageSensing_2018, slide 25-26

Exercise 4
Blooming looks lika a big white spot in the image.
Both the photodiode and the MOS capacitor collect electric charge in a small region corresponding to the conductor region. When this region becomes saturated, the charge spills over to neighboring elements.
ImageSensing_2018, slide 24

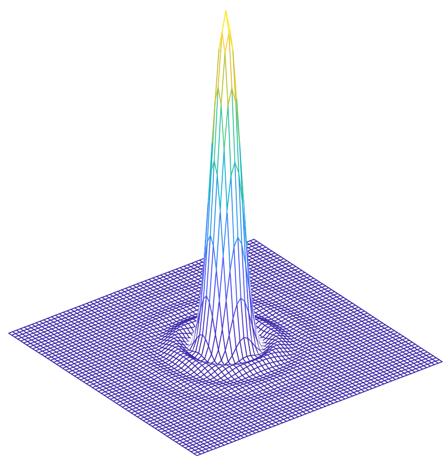
Exercise 5
Internal heat in the sensor will be detected as IR radiation, and mixed with the IR radiation from the objects in the scene.
Infrared and Multispectral Imaging, slide 34-36

Exercise 6
Glass absorbs a large range of the IR-spectrum, while germanium does not, i.e. the transmittance is better for germanium.
Infrared and Multispectral Imaging, slide 45-47

Exercise 7

a) $\Delta x \approx 1.22 \cdot \lambda \cdot f_L / D$

b)



c) The circular box $g(r) = 1$ for $r <= 0.5$ and $g(r) = 0$ for $r > 0.5$, where r is the radial distance.

d) The width Δx of the point spread function is the smallest detail that can be resolved by the optical system of the camera. This means that having a pixel size that is significantly smaller than Δx, for example by increasing the number of pixels in the sensor, does not increase the real resolution in the image.

ImageFormation_2018, slide 53-63

Exercise 8

a) The signal energy is proportional to the squared pixel intensity, $S = const_1 \cdot i^2$. The noise energy is proportional to the noise variance, which is proportional to the pixel intensity, $N = const_2 \cdot i$. Consequently, $S/N = const \cdot i$, i.e. $k = 1$.

b) There are more noise in the blue channel giving a comparably higher S/N for red and green. Additive mixing of red and green becomes yellow.

Computer exercise: The Digital Camera

PART II: GEOMETRY AND MULTIPLE VIEWS

Exercise 9

The two (straight) parallel lines in B show up as:

- Two straight parallel lines in A.
- Two straight, but not necessary parallel, lines in H.

Exercise 10
If the view directions between the reference image and the stitched image are very different, the "resolution in" and the "size of" the two images will vary a lot. An attempt to stitch an image from 90° view direction onto an image from 0° view direction will result in infinity size of the stitched image. This problem is solved by mapping the images onto a sphere and stitch them there instead.

Exercise 11
The *outer parameters* are in $[\mathbf{R}\,\mathbf{t}]$. They depend on how the camera is *oriented* in relation to the world coordinate system.

Exercise 12
If $\mathbf{C}$ is not at infinity, then we can always find a unique decomposition of $\mathbf{C}$ into its internal $\mathbf{A}$ and external $[\mathbf{Rt}]$ parameters.

Exercise 13
???? = 2377. The relation between pixel height and pixel width, $913/1132 = 1917/2377 = 0.81$, will not change during zoom.

Exercise 14
Zhang's original distorsion coefficients are k_1 and k_2. They depend on r^2 and r^4 respectively.

Exercise 15

$$\begin{pmatrix} x_n \\ y_n \\ 1 \end{pmatrix} = \mathbf{K^{-1}} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}, \quad \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} = \begin{pmatrix} x_n/\sqrt{x_n^2+y_n^2+1} \\ y_n/\sqrt{x_n^2+y_n^2+1} \\ 1/\sqrt{x_n^2+y_n^2+1} \end{pmatrix},$$

where $(x_n, y_n, 1)$ are coordinates on the normalized image plane and (x_s, y_s, z_s) are coordinates on the unit sphere.

Exercise 16
In the proof below we will use the fact that $\mathbf{r}_1 \cdot \mathbf{r}_2 = 0$ and $||\mathbf{r}_1|| = ||\mathbf{r}_2|| = 1$.

$$\left\{ \begin{array}{l} \mathbf{h}_1 = \lambda\mathbf{A}\mathbf{r}_1 \\ \mathbf{h}_2 = \lambda\mathbf{A}\mathbf{r}_2 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \mathbf{r}_1 = \lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_1 \\ \mathbf{r}_2 = \lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_2 \end{array} \right.$$

$$0 = \mathbf{r}_1 \cdot \mathbf{r}_2 = \mathbf{r}_1^T\mathbf{r}_2 = (\lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_1)^T\lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_2 = \lambda^{-2}\mathbf{h}_1^T(\mathbf{A}^{-1})^T\mathbf{A}^{-1}\mathbf{h}_2 \Rightarrow$$
$$\Rightarrow \mathbf{h}_1^T\mathbf{A}^{-T}\mathbf{A}^{-1}\mathbf{h}_2 = 0, \text{ i.e. the first constraint.}$$

$$\left\{ \begin{array}{l} 1 = ||\mathbf{r}_1||^2 = \mathbf{r}_1 \cdot \mathbf{r}_1 = \mathbf{r}_1^T\mathbf{r}_1 = (\lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_1)^T\lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_1 = \lambda^{-2}\mathbf{h}_1^T\mathbf{A}^{-T}\mathbf{A}^{-1}\mathbf{h}_1 \\ 1 = ||\mathbf{r}_2||^2 = \mathbf{r}_2 \cdot \mathbf{r}_2 = \mathbf{r}_2^T\mathbf{r}_2 = (\lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_2)^T\lambda^{-1}\mathbf{A}^{-1}\mathbf{h}_2 = \lambda^{-2}\mathbf{h}_2^T\mathbf{A}^{-T}\mathbf{A}^{-1}\mathbf{h}_2 \end{array} \right. \Rightarrow$$

$$\Rightarrow \mathbf{h}_1^T\mathbf{A}^{-T}\mathbf{A}^{-1}\mathbf{h}_1 = \mathbf{h}_2^T\mathbf{A}^{-T}\mathbf{A}^{-1}\mathbf{h}_2, \text{ i.e. the second constraint.}$$

PART III: NON-STANDARD IMAGE SENSORS

Exercise 17
Form the pixelwise average of the two pictures.
ExoticCameras_2018 Slides 4-5

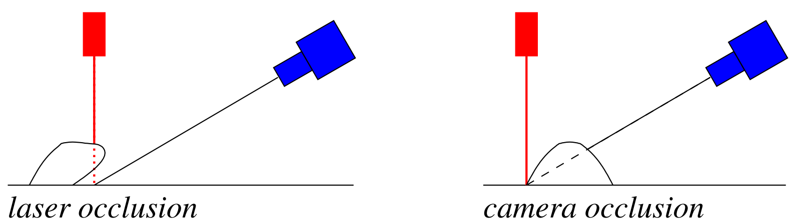
Exercise 18

- Which one is measured in advance and known by the CT-scanner? $I_0(r,\theta)$
- Which one is measured by the CT-scanner during the examination? $I(r,\theta)$
- Which one is computed by the CT-scanner as an intermediate result? $p(r,\theta)$
- Which one computed by the reconstruction algorithm as the final output from the CT-scanner? $\mu(x,y)$

Exercise 19
C_t is the color or intensity of voxel number t and α_t is the opacity of voxel number t.

Exercise 20
The second principle is *Light pulse and time measurement*.
A light pulse is send out and the time t [s] it takes for it to come back is measured. Since $s = v \cdot t$ and $v = c = 3 \cdot 10^8$ m/s, the distance is $s/2 = c \cdot t/2$ [m].

Exercise 21



Exercise 22
Kinect uses an IR-light dot pattern that appears to be located randomly. The dot pattern is different in every local neighborhood. (It is designed to have as low autocorrelation as possible for shifts larger than the point size and in the interval of disparities that the system needs to deal with.) The grid pattern, on the other hand, repeats itself. There is a risk of choosing the wrong grid point for subsequent triangulation and range calculation, especially if the grid pattern is dense and the shape of the object is complicated.

Exercise 23
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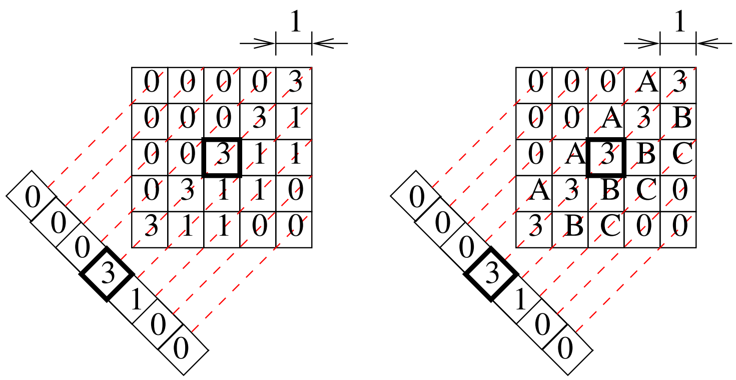
$$A = \left(1 - 1/\sqrt{2}\right) \cdot 3 \approx 0.879$$
$$B = \left(1 - 1/\sqrt{2}\right) \cdot 3 + 1/\sqrt{2} \cdot 1 \approx 1.586$$
$$C = \left(1 - (\sqrt{2} - 1)\right) \cdot 1 \approx 0.586$$



Exercise 24
For a diffusely reflecting object, the intersity I experienced by the eye is $I = const \cdot \mathbf{L} \bullet \mathbf{n}$.

$$I = const \cdot \mathbf{L} \bullet \mathbf{n} = const \cdot (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \bullet \frac{-(f_x, f_y, f_z)}{\sqrt{f_x^2 + f_y^2 + f_z^2}}$$
$$= const \cdot \frac{1}{\sqrt{2}} \cdot \frac{-(f_y + f_z)}{\sqrt{f_x^2 + f_y^2 + f_z^2}}$$