Information page for written examinations at Linköping University

Examination date	2019-04-23
Room (1)	TER3(14)
Time	14-18
Edu. code	TSBB09
Module	TEN2
Edu. code name **Module name**	Image Sensors (Bildsensorer) Written examination (Skriftlig tentamen)
Department	ISY
Number of questions in the examination	24
Teacher responsible/contact person during the exam time	Maria Magnusson
Contact number during the exam time	073 804 38 67
Visit to the examination room approximately	15.00, 17.00
Name and contact details to the course administrator (name + phone nr + mail)	Carina Lindström 013-284423 carina.e.lindstrom@liu.se
Equipment permitted	Calculator, pen and paper
Other important information	
Number of exams in the bag	

Guide

The written examination consists of 3 parts, one part for each of the three course aims in the curriculum.

- Part I: standard image sensors, including IR
- Part II: geometry and multiple views
- Part III: non-standard image sensors

Each part consists of 6 exercises where the student should demonstrate ability to explain concepts, phenomena, etc (type A exercises), and 2 additional exercises that test a deeper understanding of various topics in the course, for example, in terms of simpler calculations (type B exercises).

Type A exercises give at most 1 point each. Type B exercises give at most 2 points each.

To pass with grade 3: At least a total of 4 points each in each of the three parts.

To pass with grade 4: At least a total of 6 points each in each of the three parts.

To pass with grade 5: At least a total of 8 points each in each of the three parts.

The answers to the A-exercises should be given in the blank spaces of this examination thesis, below the questions.

The answers to the B-exercises should be given on blank paper sheets, with no more than one exercise per sheet, that will be appended to the thesis by the student.

Write your AID code at the top of the pages in this examination thesis and any sheet appended to the examination thesis. Appended sheets must also have the course code and date written on them and be numbered.

You can write your answers either in Swedish or English.

Good luck!
Maria Magnusson and Robert Forchheimer

AID code:

PART I: STANDARD & IR IMAGE SENSORS

Exercise 1 (A, 1p) **Image formation**
Which light, red or blue, gives the sharpest image? Motivate your answer by giving references to the equation $\Delta x \approx 1.22 \cdot \lambda \cdot f_L/D$. Also, explain what Δx is.

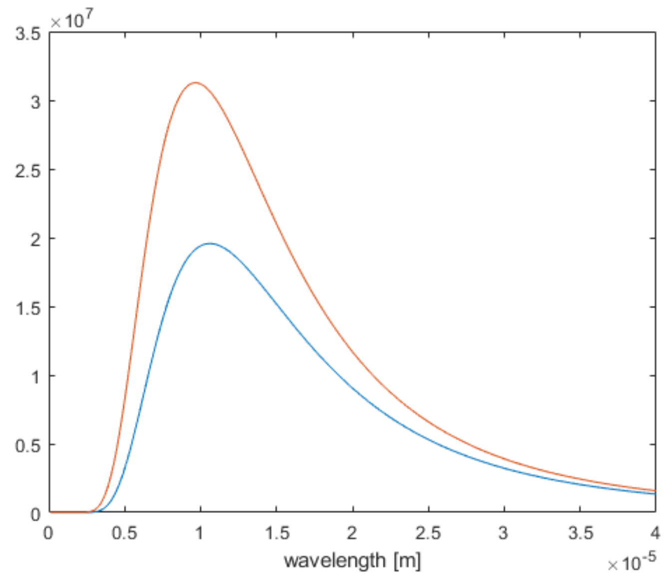
Exercise 2 (A, 1p) **Image formation**
Explain the concept of *chromatic aberration* for lenses in a camera and describe how it affects the quality of the resulting image?

Exercise 3 (A, 1p) **Image sensors**
Why is it important that a sensor chip has a high *fill factor*?

Exercise 4 (A, 1p) **Infrared and Multispectral Imaging**
What do SWIR, MWIR, and LWIR stand for?

Exercise 5 (A, 1p) **Infrared and Multispectral Imaging**
Explain the concept of a *multi-spectral image sensor*.

Exercise 6 (A, 1p) **Infrared and Multispectral Imaging**
In the computer exercise related to infra-red sensors you were acquainted with functions that are plotted in the figure below.
What is the physical entity on the vertical axis?
What is the physical difference between the two curves?



Exercise 7 (B, 2p) **Image sensors. Shading correction.**
Assume that you have two reference images, one dark and one bright, given by $b_A(x, y)$ and $b_B(x, y)$, respectively. Also assume that you have an original image $b(x, y)$ taken with the camera. The corrected image will be calculated as $\hat{f}(x, y) = c(x, y)\,(b(x, y) + d(x, y))$. The values in $b_A(x, y)$ should be corrected to $\hat{f}_A(x, y)$ and the values in $b_B(x, y)$ should be corrected to $\hat{f}_B(x, y)$. Note that $\hat{f}_A(x, y)$ and $\hat{f}_B(x, y)$ are contant values independent of (x, y).
Express c and d as functions of b_A, b_B, $\hat{f}_A$ and $\hat{f}_B$.

WRITE YOUR ANSWER ON A SEPARATE SHEET

Exercise 8 (B, 2p) **Image sensors**

A small part of a Bayer image is shown below, left, with a corresponding Bayer pattern, right. Compute the numerical values of *Gimage*, the resulting green (G) color plane, for the small part of the image.
Assume that pixels outside the small part of the image are zero.
Assume that the Bayer pattern repeats itself outside the small part of the image.

0	0	0	0	0
0	0	1	0	0
0	1	2	1	0
1	2	3	2	1
2	3	3	3	2

Bayer image

R	G			
G	B			

Bayer pattern

Gimage ?

Use normalized averaging with the interpolation kernel w shown below (center is marked in boldface).

$$w = \frac{1}{4}\begin{array}{|c|}\hline 1 \\ \hline \mathbf{2} \\ \hline 1 \\ \hline\end{array} * \frac{1}{4}\begin{array}{|c|c|c|}\hline 1 & \mathbf{2} & 1 \\ \hline\end{array} = \frac{1}{16}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & \mathbf{4} & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

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PART II: GEOMETRY AND MULTIPLE VIEWS

Exercise 9 (A, 1p)
Regard an image B with two parallel lines. Transform image B to an image A with an *affine* transformation. (Such a transformation contains translation, scaling, rotation, skewing.) Moreover, transform image B to an image H with a *homography* transfomation. How do the two parallel lines in B show up in A and H? Mention both similarities and differences.

Exercise 10 (A, 1p)
In order to stitch a set of smaller images to larger panorama image, it is important that the initial set of images are taken by rotating the camera around its center. Explain what happen if the camera is translated, too.

Exercise 11 (A, 1p)
How can you use an image showing the projection of multiple lines to calibrate for the lens distortion of the camera?

Exercise 12 (A, 1p)
An advantage with Zhang's method for camera calibration is that it only requires a simple 2D calibration pattern.
An alternative to Zhang's method is to use a 3D calibration object and solve $\mathbf{C}$ in

$$s(u, v, 1)^T = \mathbf{C} \cdot (X, Y, Z, 1)^T = \mathbf{A}[\mathbf{Rt}] \cdot (X, Y, Z, 1)^T$$

It is then possible to extract $\mathbf{A}[\mathbf{Rt}]$ from $\mathbf{C}$ if desired. What is the minimum number of 3D points on the 3D calibration object in order to solve $\mathbf{C}$?

Exercise 13 (A, 1p)
The intrinsic camera parameters in terms of the matrix are

$$\mathbf{A} = \begin{pmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Why is it desirable that $\alpha = \beta$ and $\gamma = 0$?

Exercise 14 (A, 1p)
Zhang's method for camera calibration is based on the expression

$$\lambda\, \mathbf{A}\, [\, \mathbf{r}_1\, \mathbf{r}_2\, \mathbf{t}\,] = [\, \mathbf{h}_1\, \mathbf{h}_2\, \mathbf{h}_3\,]\,,$$

where $[\, \mathbf{h}_1\, \mathbf{h}_2\, \mathbf{h}_3\,]$ is the measured homography for one view of the calibration pattern. This expression allows us to formulate two constraints on $\mathbf{A}$, where one constraint is derived from the relation $\mathbf{r}_1 \cdot \mathbf{r}_2 = 0$. Which is the other relation that leads to a constraint on $\mathbf{A}$?

Exercise 15 (B, 2p)
In the computer exercise on constructing panorama images, we performed image stitching in spherical coordinates as:

a) A number of corresponding points in the two images were detected and transformed to the unit sphere.

b) ???

c) The two images were resampled to correct spherical coordinates.

In a), how were homogeneous image coordinates $(u, v, 1)$ transformed to coordinates on the 3D unit sphere (1p)?
Hint: Use the camera calibration matrix $\mathbf{A}$ (denoted $\mathbf{K}$ in the panorama exercise) in your explanation!
What was performed in b) and why was this done (1p)?

Exercise 16 (B, 2p)
The external parameters of a specific camera are given as

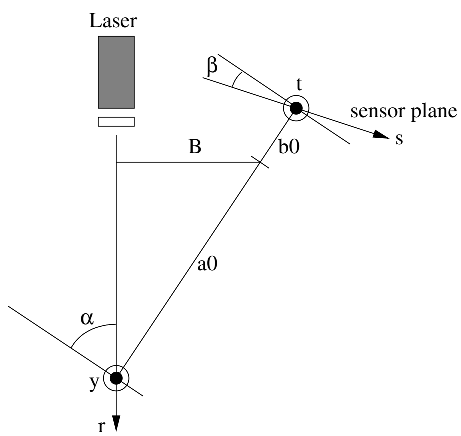
$$[\,\mathbf{R} \,|\, \mathbf{t}\,] = \begin{pmatrix} 0.5 & 0.866 & 0 & 20 \\ -0.866 & 0.5 & 0 & 35 \\ 0 & 0 & 1 & 75 \end{pmatrix}.$$

How is the principal axis (optical axis) of the camera oriented relative to the world coordinate system (1p)? Where is the world origin located in the camera coordinate system (1p)?

PART III: NON-STANDARD IMAGE SENSORS

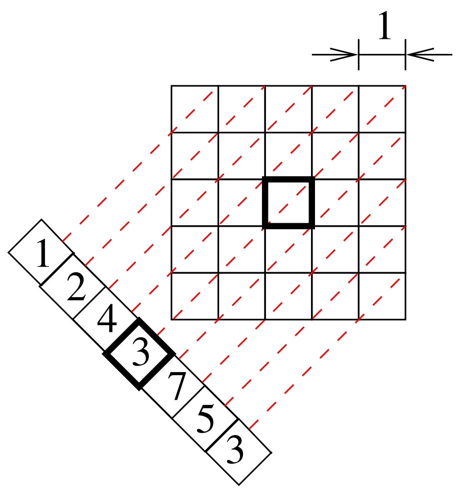
Exercise 17 (A, 1p) **Range cameras**
For a *time-of-flight* camera, the time difference was measured to 11 ns. Calculate the distance!

Exercise 18 (A, 1p) **Range cameras**
A common setup for a sheet-of-light range camera is shown in the figure below. What is the meaning of the equation $a_0 \tan\beta = b_0 \tan\alpha$?

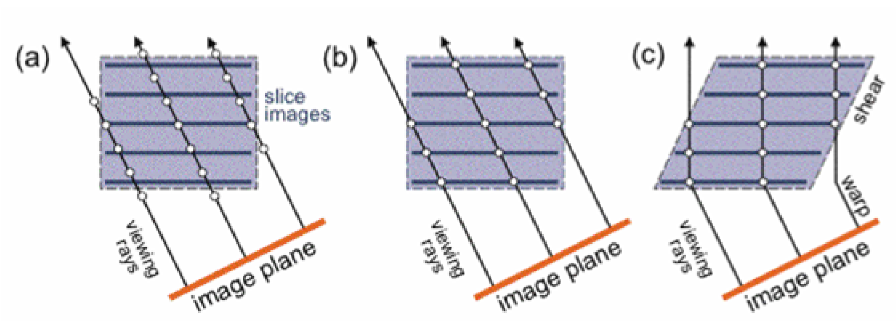


Exercise 19 (A, 1p) **CT**
The *filtered back-projection* method for tomography image reconstruction applies a ramp-filter on the detected signal $s(x)$ in each projection before it is back-projected to reconstruct the image. The filtering operation can be implemented as a multiplication in the frequency domain. To avoid artifacts, it is necessary to zero-pad $s(x)$ before it is Fourier transformed. What is the minimum amount of zero-padding?

Exercise 20 (A, 1p) **CT**
Perform backprojection of the slanted projection over the little 5 × 5-image. Use nearest neighbor interpolation. The red dashed lines are just help-lines.



Exercise 21 (A, 1p) **3D visualization**
The figure is taken from one of the lectures. It describes ray-casting. What is the advantage to perform the operation as in b) or c) compared to a)?



Exercise 22 (A, 1p) **3D visualization**
This equation is used in the context of 3D-visualization:

$$\mathbf{I}_{Phong} = k_d \mathbf{I} \cos\phi + k_s \mathbf{I} \cos^n \rho.$$

How do you set the parameters when you want your surface to look similar to *'a polished steel thermos'*, *'household paper'* or *'a perfect mirror'*? The answer should make clear how the magnitudes of the parameters (k_d, k_s, n) are different for the different materials.

Exercise 23 (B, 2p) **Exotic cameras**
Explain how a "coded aperture" can be used to extend the depth-of-field of a camera. Which mathematical operation is used to reconstruct a sharp image? Especially, explain how the equation $|f_k * x - y|^2$ is utilized.

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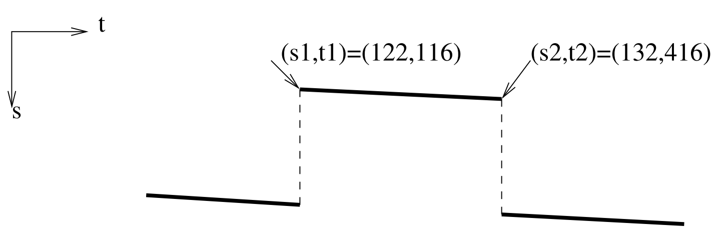
Exercise 24 (B, 2p) **Range cameras**
A range camera based on sheet-of-light and triangulation has a calibration matrix $\mathbf{C}$ that describes the relation between the sensor coordinates (s, t), measured in pixels, and the real coordinates (r, y), measured in mm, according to

$$k \cdot (s, t, 1)^T = \mathbf{C}(r, y, 1)^T.$$

By a calibration procedure, $\mathbf{C}$ has been determined to

$$\mathbf{C} = \begin{pmatrix} 2.660 & -0.683 & 53.8 \\ 0.181 & -1.440 & 174.0 \\ 0.000 & -0.002 & 1.0 \end{pmatrix}, \text{ giving } \mathbf{C}^{-1} = \begin{pmatrix} 0.3899 & -0.2055 & 14.7720 \\ 0.0646 & -0.9498 & 161.7891 \\ 0.0001 & -0.0019 & 1.3236 \end{pmatrix}.$$

A box with a parallel-trapezoidal cross-section is measured by the range camera and gives the following laser profile on the sensor. Determine the distance (in mm) between the points on the box that are represented by (s_1, t_2) and (s_2, t_2) in the image.



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