Guide to answers for distance examination in TSBB09 Image Sensors, 2021-03-15

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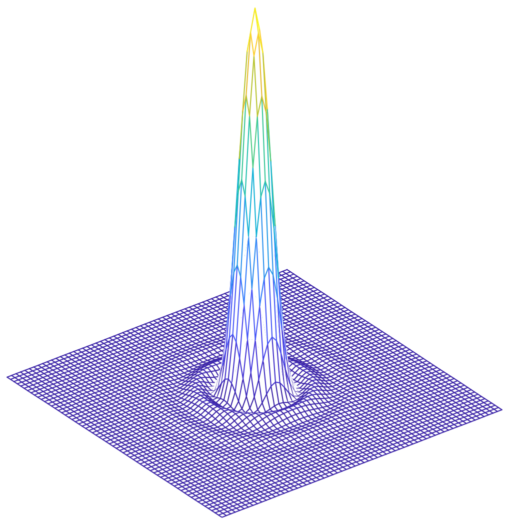
PART I: STANDARD CAMERAS & IR SENSORS

Exercise 1
The PSF is shown in the figure. It takes the shape of the Airy disk function $G^2(k\rho)$, where $G(\rho)$ is sometimes called the jinc function.



The out-of-focus PSF takes the shape of the camera aperture. For a circular aperture, the PSF is a disk, which is sometimes referred to as the circle of confusion.

Exercise 2
$\Delta x \approx 1.22 \cdot \lambda \cdot f_L/D$, where λ is the wavelength, f_L is the focal length and D is the diameter of the circular aperture. All are measured in mm, m or a similar unit.

Exercise 3
Blooming looks like a big white spot in the image.

Exercise 4
Both the photodiode and the MOS capacitor collect electric charge in a small region corresponding to the conductor region. When this region becomes saturated, the charge spills over to neighboring elements.

Exercise 5
Below 2nd order correction is described. Three blackbody references at three different temperatures are used. An algorithm including polynomial fitting gives correction equations for each pixel. After applying the correction, all pixels values correspond to the correct temperature – at least for all three blackbody references.

Exercise 6
A dead pixel has no temporal noise.
A normal pixel has medium temporal noise.
A bad pixel has high temporal noise.

Exercise 7

a)
$$\texttt{imm} = \left(\sum_{k=1}^{100} \texttt{im}_\texttt{k}\right)/100, \quad \texttt{imvar} = \left(\sum_{k=1}^{100} {\texttt{im}_\texttt{k}}^2\right)/100 - \texttt{imm}^2.$$

b) The signal energy is proportional to the squared pixel intensity,
$S = const_1 \cdot \texttt{imm}^2$.
The noise energy is proportional to the noise variance,
$N = const_2 \cdot \texttt{imvar}$.
Consequently,
$S/N = const \cdot \texttt{imm}^2/\texttt{imvar}$.

c) There are probably more noise in the blue channel giving a comparably higher S/N for red and green. Additive mixing of red and green becomes yellow.

d) In areas of the image with very high intensity, the pixel values are saturated. Then the variance is 0.

Exercise 8

a) Plank's radiation law for blackbodies.

b) Spectral radiant emittance of the blackbody.

c) Wavelength, λ, measured in nm.

d) Stefan-Bolzmann law.

PART II: GEOMETRY AND MULTIPLE VIEWS

Exercise 9
(u_0, v_0) is the cross-section between the optical axis and the real image plane.

Exercise 10
The angle is $30°$, since $\arccos(0.866) \cdot 180/\pi = 30°$. Call the camera coordinate system (U, V, W), where W is the optical axis. It has the same direction as the Z-axis in the world coordinate system (X, Y, Z).

Exercise 11
The procedure is called camera resectioning. The disadvantage is that it is much more complicated to manifacture a 3D calibration object, compared to a 2D calibration object.

Exercise 12
We want to solve for $\mathbf{A}$. $[\,\mathbf{h}_1\,\mathbf{h}_2\,\mathbf{h}_3\,]$ have been determined. $\mathbf{r}_1$ and $\mathbf{r}_2$ are unknown. Therefore, it is necessary to get rid of them.

Exercise 13
Perform the matlab-command:
```
[V D] = eig(R);
```
Then

$$\mathbf{D} = \begin{pmatrix} 0.9697 + j0.244 & 0 & 0 \\ 0 & 0.9697 - j0.244 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is recieved. The diagonal elements are the eigenvalues. Let α denote the rotation angle. The eigenvalue 1 corresponds to the rotation axis. The other eigenvalues are $e^{i\alpha}$ and $e^{-i\alpha}$, respectively. Therefore $\alpha = 14.1°$ is the angle between the two images.

Alternatively, Rodriges formula can be used. Then $\cos\alpha = 0.5(\mathrm{tr}(\mathbf{R}) - 1)$.

Exercise 14
Use a blending weight function `alpha` that has smaller values at the edges of the image and larger values at the center. `Pano1` and `Pano2` are the two images transformed to the reference grid. `alpha1` and `alpha2` are the weight image (`alpha`) transformed to the reference grid. Perform normalized weighting:

$$\texttt{Pano} = \frac{\texttt{alpha1} \cdot \texttt{Pano1} + \texttt{alpha2} \cdot \texttt{Pano2}}{\texttt{alpha1} + \texttt{alpha2}}$$

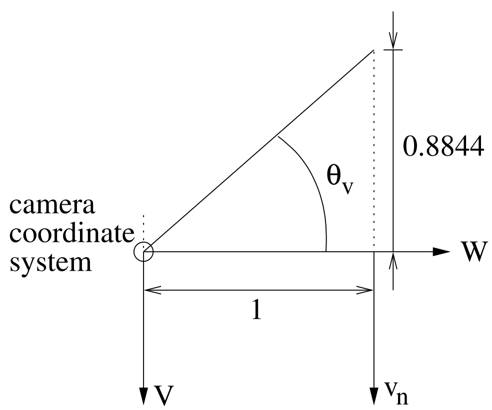
This way the borders of the individual images will be invisible in the panorama.

Exercise 15

The normalized image coordinate (u_n, v_n) is transformered to the real image coordinate $(u, v) = (277, 117)$ according to

$$\begin{pmatrix} 277 \\ 117 \\ 1 \end{pmatrix} = \begin{pmatrix} 355 & 0 & 500 \\ 0 & 320 & 400 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} u_n \\ v_n \\ 1 \end{pmatrix}.$$

Solution to this equation system gives $(u_n, v_n) = (-0.6282, -0.8844)$. Therefore, $\arctan(-0.6282/1)\text{rad} = \arctan(-0.6282)\cdot(180/\pi)^\circ \approx -32.1^\circ \Rightarrow \theta_u = 32.1^\circ$ and $\arctan(-0.8844/1)\text{rad} = \arctan(-0.8844)\cdot(180/\pi)^\circ \approx -41.5^\circ \Rightarrow \theta_v = 41.5^\circ$, see figure.



Exercise 16

$$\begin{aligned}
(r, y, 1)^T &= k\mathbf{C}^{-1}(s, t, 1)^T \\
(r_1, y_1, 1)^T &= k\mathbf{C}^{-1}(s_1, t_1, 1)^T = k\mathbf{C}^{-1}(61, 58, 1)^T \\
(r_1, y_1, 1)^T &= k(33.1775, 38.7780, 1)^T \Rightarrow (r_1, y_1) = (33.1775, 38.7780) \\
(r_2, y_2, 1)^T &= k\mathbf{C}^{-1}(s_2, t_2, 1)^T = k\mathbf{C}^{-1}(66, 208, 1)^T \\
(r_2, y_2, 1)^T &= k(-1.9396, -123.6886, 1)^T \Rightarrow (r_2, y_2) = (-1.9396, -123.6886)
\end{aligned}$$

$$\text{Answer: width} = \sqrt{(r_1 - r_2)^2 + (y_1 - y_2)^2} = 166.2185 \text{ mm} \approx 166 \text{ mm}.$$

PART III: NON-STANDARD IMAGE SENSORS

Exercise 17
Assume perfect red light: [R G B]=[1 0 0].
A black object reflects no light: [1 0 0]·[0 0 0]=[0 0 0]. Problem!
A yellow object reflects red and green light: [1 0 0]·[1 1 0]=[1 0 0]. Ok!
A white object reflects red, green and blue light: [1 0 0]·[1 1 1]=[1 0 0]. Ok!
A blue object reflects blue light: [1 0 0]·[0 0 1]=[0 0 0]. Problem!
A red object reflects red light: [1 0 0]·[1 0 0]=[1 0 0]. Ok!

Exercise 18
If the reflected laser pulse comes from a border area which is dark on one side and bright on the other side, the detected pulse shape becomes distorted. Then there will be a slight change in detected max position which will show up as a height difference in the range image. Consequently, the range deviations will occur along the border around the M letter as well as on both sides of the surrounding rectangle

Exercise 19
The algorithm should be as follows:

1) Guess correspondences using current best guess of registration.
2) Calculate the means of the two point sets, $\mathbf{X}_\mu$ and $\mathbf{Y}_\mu$. Subtract the means from the two point sets. This way, translation is removed.
3) Estimate the *rotation* $\mathbf{R}$ with the solution to the
'Orthogonal Procrustes Problem' (OPP).
4) Update the translation as $\mathbf{t} = \mathbf{X}_\mu - \mathbf{R}\mathbf{Y}_\mu$.
5) Update correspondences, and goto 2.

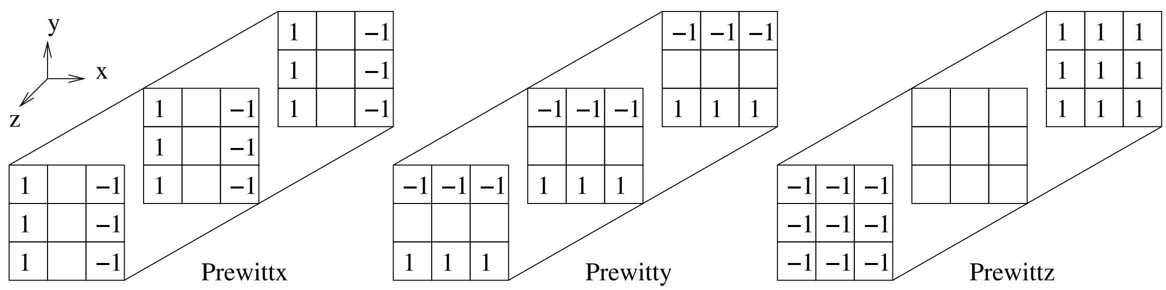
It is important the the translation is removed before Procrustes algorithm because it can only estimate rotation between two point sets with a common origin.

Exercise 20
The materials are ordered with increasing n:
rubber, plastic, chrome.
The higher n, the more shiny object, and the narrower specular lobe. (A perfect mirror has $n = \infty$.)

Exercise 21
An event camera has significantly better temporal resolution and consequently less motion blur than a standard camera. In an event camera, the pixels operate independently and asynchrounously. Only those pixels that have changed needs to be updated at every time position. In a standard camera, the whole frame is updated at every time position.

Exercise 22
Eah pixel contains two photo diodes, which have different sensitivity to light. The high-sensitivity diode operates well in dark areas and the low-sensitivity diode operates well in bright areas. Their response is combined, e.g. by computing their mean.

Exercise 23



$$\frac{\partial f(x,y,z)}{\partial x} = f(x,y,x) * \text{Prewittx},$$
$$\frac{\partial f(x,y,z)}{\partial y} = f(x,y,x) * \text{Prewitty},$$
$$\frac{\partial f(x,y,z)}{\partial z} = f(x,y,x) * \text{Prewittz}.$$

$$\text{grad}(x,y,z) = (\frac{\partial f(x,y,z)}{\partial x}, \frac{\partial f(x,y,z)}{\partial y}, \frac{\partial f(x,y,z)}{\partial z})$$
$$\bar{n}(x,y,z) = -(\text{grad}(x,y,z)/|\text{grad}(x,y,z)|)$$

Exercise 24

a) Light pulse and time measurement.

b) LIDAR = light+RADAR
Light waves have shorter wavelength than radio waves. RADAR uses short radio waves.

c) The Velodyne sensor rotates 360° and collects depth data with a 64-line ToF sensor.