Guide to answers for written examination in TSBB09 Image Sensors, 2018-04-03

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PART I: STANDARD CAMERAS & IR SENSORS

Exercise 1 See Lecture A, slides 35+36

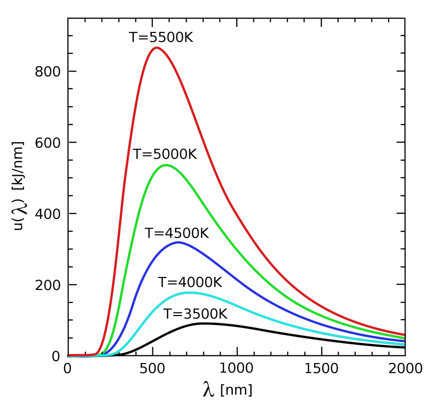
Exercise 2 See Lecture A, slides 49+50

Exercise 3 See Lecture B, slides 6

Exercise 4 In general: variations over the image array in the gain or offset parameters. In the extreme case, this effect can imply that an isolated pixel has a value that is either very low or very high compared to its neighbors, and almost independent of the incident light. See also computer exercise B.

Exercise 5 A multi-spectral camera can be implemented as a line camera, where each pixel measures the intensities in multiples wavelength bands, e.g., by splitting the light with a prism. By moving the line camera over a surface, e.g., the surface of the earth, using a satellite or an airplane, a 2D image is generated by the push-broom principle. This image is then a multi-spectral image of the surface.

Exercise 6 Each object emits radiation for all wavelengths, but with different energy for different wavelengths according to a function that is characteristic for the temperature of the object, in accordance with Planck's radiation law. Examples of these functions for different temperatures are illustrated below. Note that the energy always increase with higher temperature, and that the function has a peak for a wavelength that becomes shorter for higher temperature.



Exercise 7 The optical resolution of the sensor, besides the general quality of the optical system, depends on the size of the aperture which refracts the light that passes through the aperture. The refraction effect is inversely proportional to the diameter of the aperture, and is manifested in terms of the point-spread function which describes the image of a point. See lecture A, slides $64 - 74$. The width Δx of the point spread function is the smallest detail that can be resolved by the optical system of the camera. This means that having a pixel size that is significantly smaller than Δx, for example by increasing the number of pixels in the sensor, does not increase the real resolution in the image.

Exercise 8

- $\mathrm{SNR} = S/N = 5^2/0.05 = 500$.
- For shot noise, the variance is proportional to the signal strength.
  $\mathrm{var}(i1) = k_1 \cdot \mathrm{mean}(i1) \Rightarrow k_1 = 0.05/5 = 0.01$.
  $\mathrm{var}(i2) = k_2 \cdot \mathrm{mean}(i2) \Rightarrow k_2 = 0.05/15 = 0.0033$.
  $\mathrm{var}(i3) = k_3 \cdot \mathrm{mean}(i3) \Rightarrow k_3 = 0.25/25 = 0.01$.
  Consequently, the measurement for pixel 2 is wrong because it gives a discrepant proportionality constant.

PART II: GEOMETRY AND MULTIPLE VIEWS

Exercise 9 An epipolar point is the projection of the center of one camera in the other camera's image. See lecture F, slides $11 - 12$.

Exercise 10 The rectifying homographies must satisfy $\mathbf{F} = \mathbf{H}_1^T \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{H}_2$, where $\mathbf{F}$ is the fundamental matrix.

Exercise 11 See lecture F, slides 59 - 61.

Exercise 12 A lens or a lens system can never map straight lines in the 3D scene exactly to straight lines in the image plane. Depending on the lens type, a square pattern will typically appear like a barrel or a pincushion.

Exercise 13 $\mathbf{H} = \mathbf{K}\,\mathbf{R}\,\mathbf{K}^{-1}$. See lecture X.

Exercise 14 $\mathbf{r}_1 \cdot \mathbf{r}_2 = 0$

Exercise 15 The different parameters have the following interpretations:

- κ_{11} is the scaling in the horizontal direction.
- κ_{22} is the scaling in the vertical direction.
- κ_{12} is the skewing, which is often close to 0.
- $(\kappa_{13}, \kappa_{23})$ is the cross-section between the optical axis and the real image plane, measured in pixels.

Exercise 16

$$\begin{pmatrix} x_n \\ y_n \\ 1 \end{pmatrix} = \mathbf{K^{-1}} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \quad , \quad \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} = \begin{pmatrix} x_n/\sqrt{x_n^2 + y_n^2 + 1} \\ y_n/\sqrt{x_n^2 + y_n^2 + 1} \\ z_n/\sqrt{x_n^2 + y_n^2 + 1} \end{pmatrix} ,$$

where $(x_n, y_n, 1)$ are coordinates on the normalized image plane and (x_n, y_s, z_s) are coordinates on the unit sphere.
The transformation is performed for two images and their respective coordinates on the unit sphere are send to Procrustes algorithm, which determines the rotation between the two images.

PART III: NON-STANDARD IMAGE SENSORS

Exercise 17 For example: A set of several cameras mounted close to each other (approximately coinciding centers) and point in different directions. A single camera with a fish-eye lens or combined with a system of lenses and mirrors that can project an extremely large field of view onto the camera image. See lecture J, slides 34 -.

Exercise 18 See the figure. The literature on range cameras with laser line suggest five methods to determine the position of the laser line. Choose one of the b)-e) methods below.

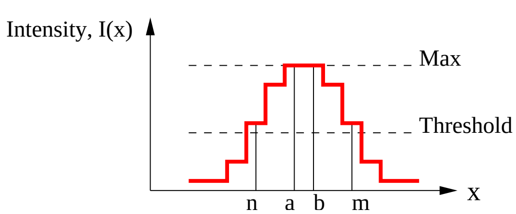
a) $pos = (a + b)/2$

b) $pos = (n + m)/2$

c) $pos = [\sum xI(x)]/[\sum I(x)]$

d) Derivate and search for the zero-crossing

e) Sub-pixel correlation with a Gaussian function



Exercise 19 A homography.

Exercise 20 Kinect uses an IR-light dot pattern which is different in every local neighborhood. It is designed to have as low autocorrelation as possible for shifts larger than the point size and in the interval of disparities that the system needs to deal with. The dot pattern in the exercise, on the other hand, repeats itself. There is a risk of choosing the wrong position for subsequent triangulation and range calculation.

Exercise 21 If $\mathbf{R} \cdot \mathbf{V}$ is negative, the ray is reflected away from the viewer. The intensity contribution $I_{specular}$ is then set to zero.

Exercise 22 I_0 is known, i.e. it is measured in advance.
$I(r, \theta)$ is measured.
$\mu(x, y)$ is computed by the reconstruction algorithm as the output from the CT-scanner.

Exercise 23 See figure. For diffuse reflection the reflected intensity is

$$\mathbf{I}_{diffuse} = \mathbf{M}\ \mathbf{I}_{light}\ (\mathbf{L} \cdot \mathbf{n}),$$

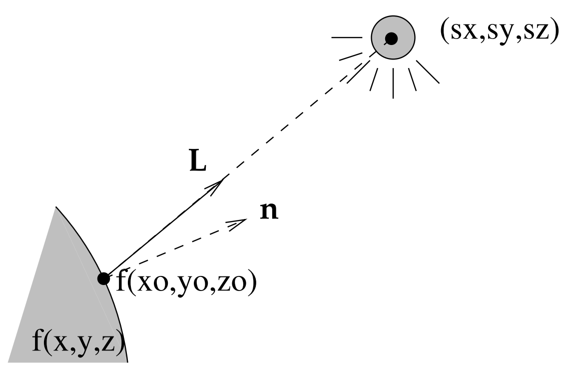
where $\mathbf{n}$ is the unit vector of the surface normal given by

$$\mathbf{n} = -\frac{\left(\frac{\partial f(x,y,z)}{\partial x}, \frac{\partial f(x,y,z)}{\partial y}, \frac{\partial f(x,y,z)}{\partial z}\right)}{\sqrt{\frac{\partial f(x,y,z)}{\partial x}^2 + \frac{\partial f(x,y,z)}{\partial y}^2 + \frac{\partial f(x,y,z)}{\partial z}^2}}$$

and where $\mathbf{L}$ is the unit vector to the light source given by

$$\mathbf{L} = \frac{(s_x - x_0, s_y - y_0, s_z - z_0)}{\sqrt{(s_x - x_0)^2, (s_y - y_0)^2, (s_z - z_0)^2}}$$

The additional information needed is the intensity of the light $\mathbf{I}_{light}$, possibly in color. (The intensity decreases proportionally to the distance squared.) Also the color of the surface $\mathbf{M}$ is needed.



Exercise 24

a) Consider $-1 < x < 1$. Then

$$p(x,0) = \int_{-\infty}^{\infty} f(x,y)\, dy = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 0.7\, dy = 0.7 \cdot 2\sqrt{1-x^2} = 1.4 \cdot \sqrt{1-x^2}.$$

Consequently,

$$p(r,0) = \begin{cases} 1.4\sqrt{1-r^2}, & |r| \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Since the object function $f(x,y)$ has radial symmetry, the projection data will be the same for all angles, i.e.

$$p(r,\theta) = \begin{cases} 1.4\sqrt{1-r^2}, & |r| \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

b)