Information page for written examinations at Linköping University

Examination date	2019-01-16
Room (2)	U6(20) U7(13)
Time	8-12
Edu. code	TSBB09
Module	TEN2
Edu. code name **Module name**	Image Sensors (Bildsensorer) Written examination (Skriftlig tentamen)
Department	ISY
Number of questions in the examination	24
Teacher responsible/contact person during the exam time	Maria Magnusson
Contact number during the exam time	073 804 38 67
Visit to the examination room approximately	9.00, 11.00
Name and contact details to the course administrator (name + phone nr + mail)	Carina Lindström 013-284423 carina.e.lindstrom@liu.se
Equipment permitted	Calculator, pen and paper
Other important information	
Number of exams in the bag	

Guide

The written examination consists of 3 parts, one part for each of the three course aims in the curriculum.

- Part I: standard image sensors, including IR
- Part II: geometry and multiple views
- Part III: non-standard image sensors

Each part consists of 6 exercises where the student should demonstrate ability to explain concepts, phenomena, etc (type A exercises), and 2 additional exercises that test a deeper understanding of various topics in the course, for example, in terms of simpler calculations (type B exercises).

Type A exercises give at most 1 point each. Type B exercises give at most 2 points each.

To pass with grade 3: At least a total of 4 points each in each of the three parts.

To pass with grade 4: At least a total of 6 points each in each of the three parts.

To pass with grade 5: At least a total of 8 points each in each of the three parts.

The answers to the A-exercises should be given in the blank spaces of this examination thesis, below the questions.

The answers to the B-exercises should be given on blank paper sheets, with no more than one exercise per sheet, that will be appended to the thesis by the student.

Write your AID code at the top of the pages in this examination thesis and any sheet appended to the examination thesis. Appended sheets must also have the course code and date written on them and be numbered.

You can write your answers either in Swedish or English.

Good luck!
Maria Magnusson and Robert Forchheimer

AID code:

PART I: STANDARD & IR IMAGE SENSORS

Exercise 1 (A, 1p) **Image formation**
When visible light interacts with matter (normally) some fraction α of it is absorbed, some fraction ρ is reflected, and some fraction τ is transmitted. How are α, ρ and τ related? Give an equation!

Exercise 2 (A, 1p) **Image formation**
Vignetting is the "darkening" of an image towards its edges and corners. Which simple mathematical function closely approximates this phenomenon? Also explain or sketch the variable of the function.

Exercise 3 (A, 1p) **Image sensors**
Describe the concepts *fill factor* and *micro lenses* and explain how they are related.

Exercise 4 (A, 1p) **Image sensors**
How does *blooming* manifests itself in an image? What happens electronically when *blooming* occur?

AID code:

Exercise 5 (A, 1p) **Infrared and Multispectral Imaging**
High quality cameras in the infra-red range are often equipped with a cooling system. Why does this give a better image quality?

Exercise 6 (A, 1p) **Infrared and Multispectral Imaging**
Why is the optics of a IR-sensor made of rather expensive materials, such as germanium, instead if glass?

Exercise 7 (B, 2p) **Image formation**
The resolution of a camera in terms of the smallest detail that can be resolved, is limited and mainly determined by 2 factors. The first factor is the point-spread function, which is related to the optical system. It is sometimes called the airy disk. Its equation can be written $G^2(k\rho)$, where k is a constant and ρ is the radial distance. The second factor has to do with the number of detector elements (pixels) per millimeter in the sensor chip.

a) The width Δx of the point-spread function Δx, is given by an equation $\Delta x \approx 1.22...$
Complete this equation with the lens focal length f_L, the lens diameter (or aperture) D, and the light wavelength λ.

b) Sketch the airy disk function $G^2(k\rho)$ approximately.

c) $G(\rho)$ is the Fourier transform of a function. Which one?
Tip: This was also given on the "Computed Tomography 2" lecture.

d) Discuss why the "number of pixels per millimeter" alone, is not a good measure of the camera resolution.

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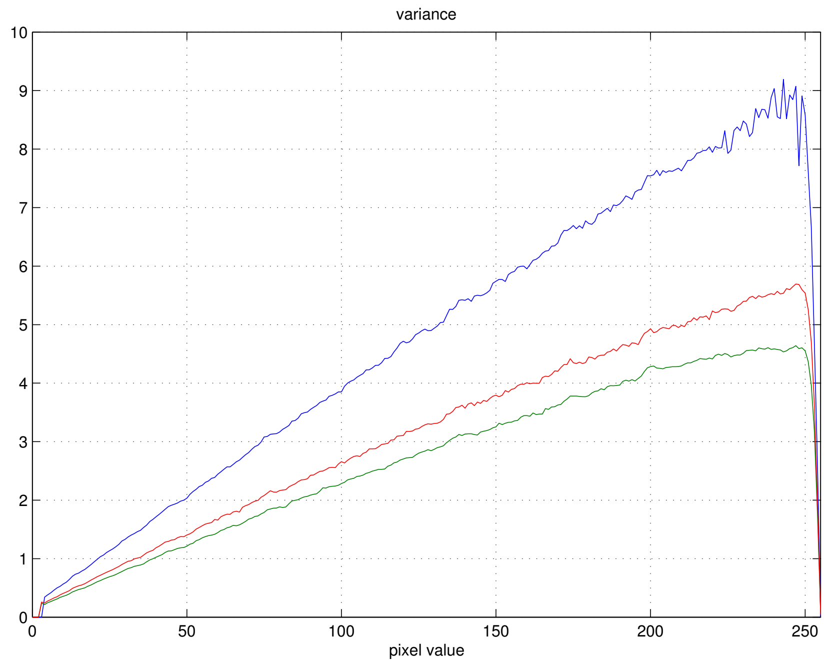
Exercise 8 (B, 2p) **Image sensors**
The figure below shows a diagram from the first computer exercise. The noise variance for the red, blue and green sensor elements, are plotted against the pixel intensity for a camera sensor.



a) The signal-to-noise ratio $S/N = const \cdot i^k$, where i is the intensity. What is k? Derive or motivate the correct answer!

b) S/N can be shown as an image. It reminds on the original image, but it is more yellowish. Explain why!

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PART II: GEOMETRY AND MULTIPLE VIEWS

Exercise 9 (A, 1p) **Geometry transformations**
Regard an image B with two parallel lines. Transform image B to an image A with an *affine* transformation. (Such a transformation contains translation, scaling, rotation, skewing.) Moreover, transform image B to an image H with a *homography* transfomation.
How do the two parallel lines in B show up in A and H? Mention both similarities and differences.

Exercise 10 (A, 1p) **Panorama**
A set of images of a large scene is taken and used to produce a large panorama image. In order to build a panorama image from images taken in many different directions it may be advantageous to first map the images to the surface of a sphere. Explain why.

Exercise 11 (A, 1p) **Camera calibration**
Consider the following expression, which describes the projection of a 3D point (X, Y, Z) to an image point (u, v):

$$s \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \mathbf{A}[\mathbf{R}\;\mathbf{t}] \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Which parameters in this expression are called *outer parameters* or *external parameters*, and why?

Exercise 12 (A, 1p) **Camera calibration**
What is camera resectioning? *Hint:* $\mathbf{C} = \mathbf{A}[\mathbf{Rt}]$.

Exercise 13 (A, 1p) **Camera calibration**
A camera with optical zoom is calibrated for two different zoom settings, leading to two intrinsic camera matrices $\mathbf{A}_1$ and $\mathbf{A}_2$,

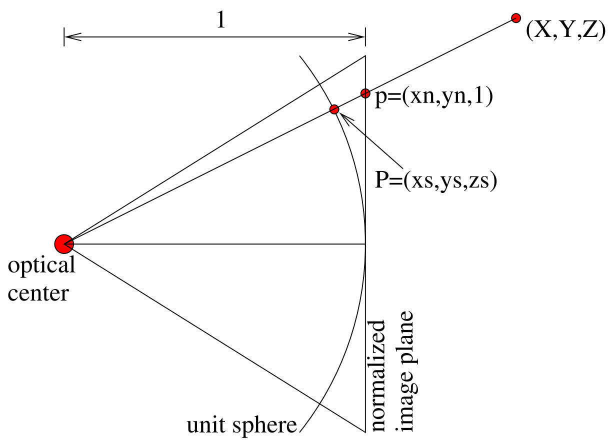
$$\mathbf{A}_1 = \begin{pmatrix} 1132 & 0 & 190 \\ 0 & 913 & 134 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{A}_2 = \begin{pmatrix} ???? & 0 & 195 \\ 0 & 1917 & 131 \\ 0 & 0 & 1 \end{pmatrix}.$$

Determine the element marked with question marks and motivate your choice.

Exercise 14 (A, 1p) **Zhang's method**
The implementation in OpenCV of Zhang's method includes a further development of Zhang's lens distortion model. The output vector of distortion coefficients is $(k_1, k_2, p_1, p_2[, k_3[, k_4, k_5, k_6]])$, where k_i are the radial and p_j the tangential distortion coefficients.
Which are Zhang's original distortion coefficients and how do they depend on the radius r?

Exercise 15 (B, 2p) **Panorama**

For panorama stitching in spherical coordinates, image points are transformed to a unit sphere and the rotation between two images is found by using Procrustes algorithm.



See the figure above. A point $(X, Y, Z)^T$ can be projected to the normalized image plane $(x_n, y_n, 1)^T$, and then transformed to a point $(u, v, 1)^T$ on the real image grid through the camera matrix $\mathbf{K}$ according to

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \mathbf{K} \begin{pmatrix} x_n \\ y_n \\ 1 \end{pmatrix} .$$

Give equations how to calculate a 3D point on the unit sphere $(x_s, y_s, z_s)^T$ from an image grid point $(u, v, 1)^T$.

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Exercise 16 (B, 2p) **Zhang's method**

Zhang's method for camera calibration is based on the expression

$$\lambda \, \mathbf{A} \, [\, \mathbf{r}_1 \; \mathbf{r}_2 \; \mathbf{t} \,] = [\, \mathbf{h}_1 \; \mathbf{h}_2 \; \mathbf{h}_3 \,] \, ,$$

where $[\, \mathbf{h}_1 \; \mathbf{h}_2 \; \mathbf{h}_3 \,]$ is the measured homography for one view of the calibration pattern and λ is a scaling parameter which implies that the left hand side is proportional to the right hand side. This expression allows us to formulate two constraints on $\mathbf{A}$,

$$\mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2 = 0$$
$$\mathbf{h}_1^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{A}^{-T} \mathbf{A}^{-1} \mathbf{h}_2$$

Use the fact that $\mathbf{r}_1$ and $\mathbf{r}_2$ are the first two columns of a rotation matrix to prove the two constraints above.

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PART III: NON-STANDARD IMAGE SENSORS

Exercise 17 (A, 1p) **Exotic Cameras**
One technique to obtain a high dynamic range (HDR) image from an ordinary camera is to make two exposures with short and long exposure time respectively. How can the two exposed images be combined to generate the HDR image?

Exercise 18 (A, 1p) **CT**
The following equations are used in computed tomography (CT):

$$I(r,\theta) = I_0(r,\theta)\exp\left(-\int_L \mu(x,y)dl\right), \qquad p(r,\theta) = \ln\left(\frac{I_0(r,\theta)}{I(r,\theta)}\right).$$

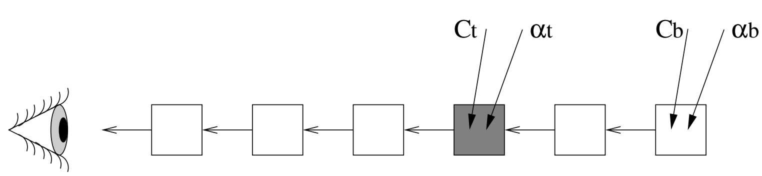
They include the functions $p(r,\theta), I(r,\theta), \mu(x,y)$ and $I_0(r,\theta)$.

- Which one is measured in advance and known by the CT-scanner? ....
- Which one is measured by the CT-scanner during the examination? ....
- Which one is computed by the CT-scanner as an intermediate result? ....
- Which one computed by the reconstruction algorithm as the final output from the CT-scanner? ....

Exercise 19 (A, 1p) **3D visualization**
In back-to-front compositing, the following iterative equations are used,

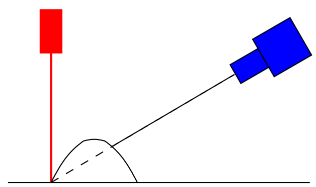
$$\begin{cases} C_b^b = \alpha_b C_b, \\ C_t^b = \alpha_t C_t + (1-\alpha_t)C_{t+1}^b. \end{cases}$$

Each voxel, at position t along the desired ray, has a corresponding value for C_t and an α_t, as illustrated in the figure. What is the physical interpretation of C_t and α_t?

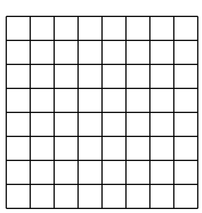


Exercise 20 (A, 1p) **Range Cameras**
One approach to active range cameras is *Time-of-flight*, which can be implemented based on two different principles. One principle is *Amplitude modulated light and phase shift measurement*. Describe the other principle. In your description, an equation for the calculated range should be given.

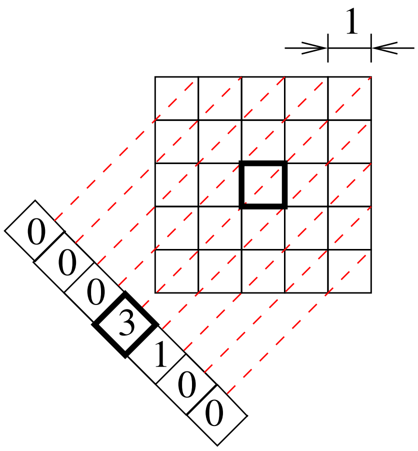
Exercise 21 (A, 1p) **Range Cameras**
The image below illustrates the case of *camera occlusion*. Draw an image to the right that illustrates *laser occlusion*.

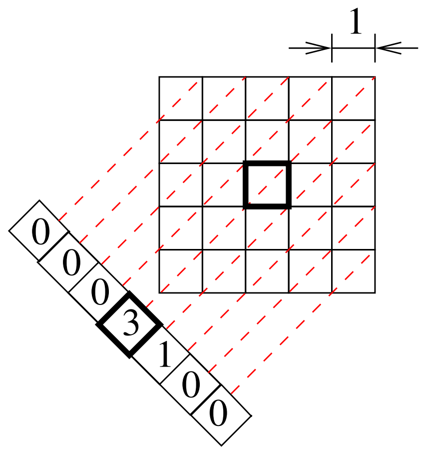


Exercise 22 (A, 1p) **Range Cameras**
One type of range camera illuminates the scene/object with a grid pattern, see the figure below. Kinect Version 1 uses another pattern. Describe the characteristic properties of this pattern and describe the main advantage of the Kinect pattern compared to the grid pattern.



Exercise 23 (B, 2p) **CT**
See the figure. The slanted projection, with the values [0, 0, 0, 3, 1, 0, 0] will be backprojected over the little 5×5-image. The red dashed lines are just help-lines.





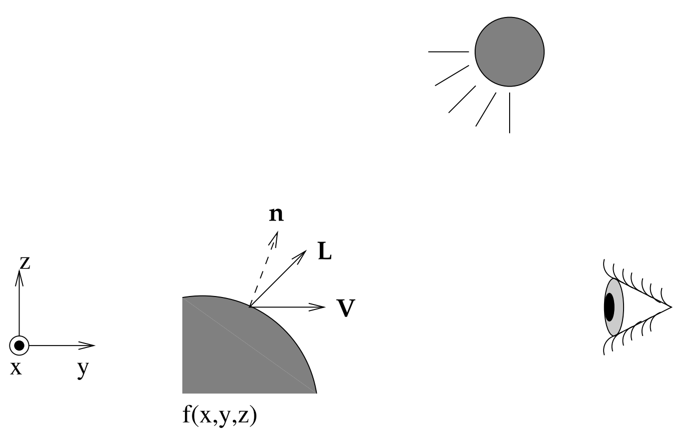
a) Use nearest neighbor interpolation. Show the result in the figure to the left!

b) Use linear interpolation with $\Lambda(x)$,

$$\Lambda(x) = \begin{cases} 1 - |x|, & \text{for } -1 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Show the result in the figure to the right!

Exercise 24 (B, 2p) **3D visualization**
The figure below illustrates a diffusely reflecting object, represented by the known density function $f(x, y, z)$. The direction to the light source is $\mathbf{L} = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and to the eye it is $\mathbf{V} = (0, 1, 0)$. Calculate the intensity experienced by the eye for the given geometry at the indicated point. The answer should be simplified to only contain the variables $(f_x, f_y, f_z) = (\frac{\partial f(x,y,z)}{\partial x}, \frac{\partial f(x,y,z)}{\partial y}, \frac{\partial f(x,y,z)}{\partial z})$. For simplicity, let the color of the light and material be white, $\mathbf{I} = 1$ and $\mathbf{M} = 1$.



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