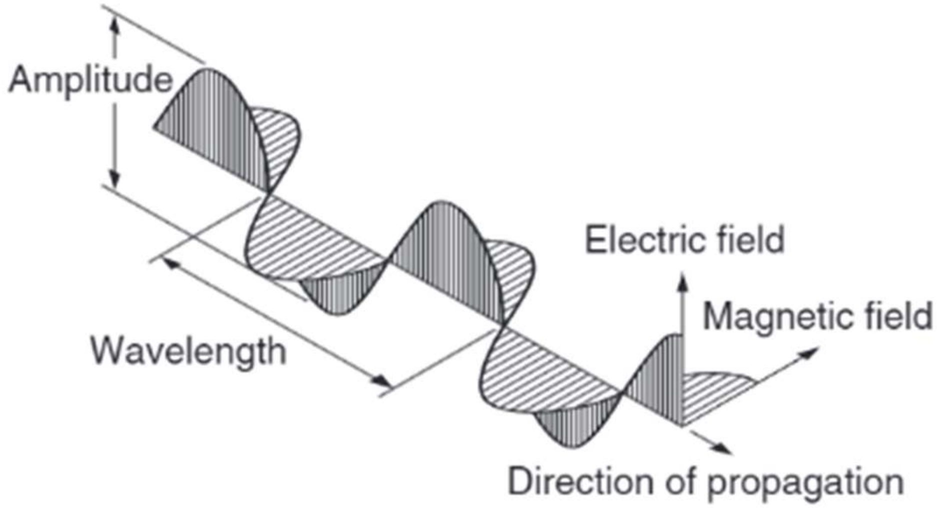
TSBB21, Lecture 1
Image Formation

- Basic physics
  - Frequency and wavelength
  - Particles and energy
- Spectrum
- Polarization
- Coherence
- Radiometry
- Interaction between light/matter
  - Refraction
  - Absorption
  - Color and absorption
  - Surface reflection
  - Emission
  - Scattering
- The plenoptic function
- The pinhole camera
  - The virtual image plane
  - The perspective projection
- Lenses
- Topics we will return to and deepen

- Thanks to:
  - Gonzales & Woods: Digital Image Processing, Global Edition, 4th edition, a few images.
  - Klas Nordberg, who initiated this course. Many slides in this lecture are similar to his slides.

Basic physics

- Electromagnetic radiation consists of electromagnetic waves
  - They have energy
  - They propagate through space
- The waves consist of *transversal* electrical and magnetic fields that alternate with a temporal frequency ν (Hertz) and spatial wavelength λ (meter)



Basic physics: Frequency and wavelength

- The relation between frequency and wavelength is:

$$c = \lambda\nu$$

  where c is the speed of light and depends on the medium, $c \leq c_0$
- c_0 = speed of light in vacuum ≈ $3 \cdot 10^8$ m/s

Basic physics:
Particles and energy

- Light can also be represented as particles, *photons*

- The energy of a photon is:

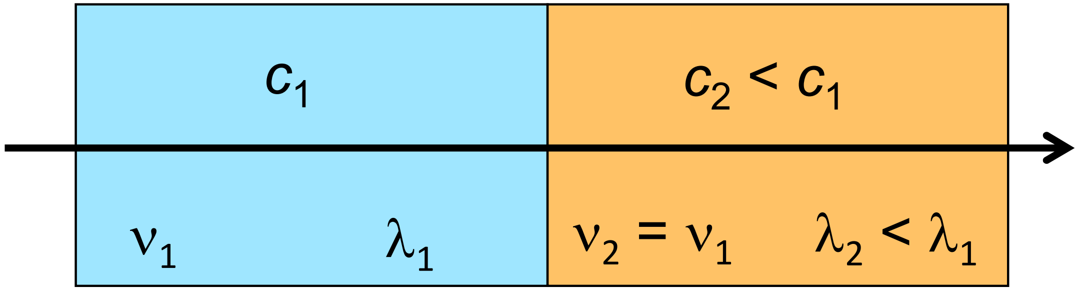
$$E = h\,\nu = h\,c\,/\,\lambda$$

Note that energy increases with ν and decreases with λ

where h is Planck's constant ($\approx 6.623 \cdot 10^{-34}$ Js)

Basic physics: Particles and energy

- As mentioned previously, energy depends on the frequency ν ($E = h\nu = h\,c\,/\,\lambda$)
- Energy is preserved when traveling from one medium to another



c_2 and λ_2 must change with the same factor relative to c_1 and λ_1

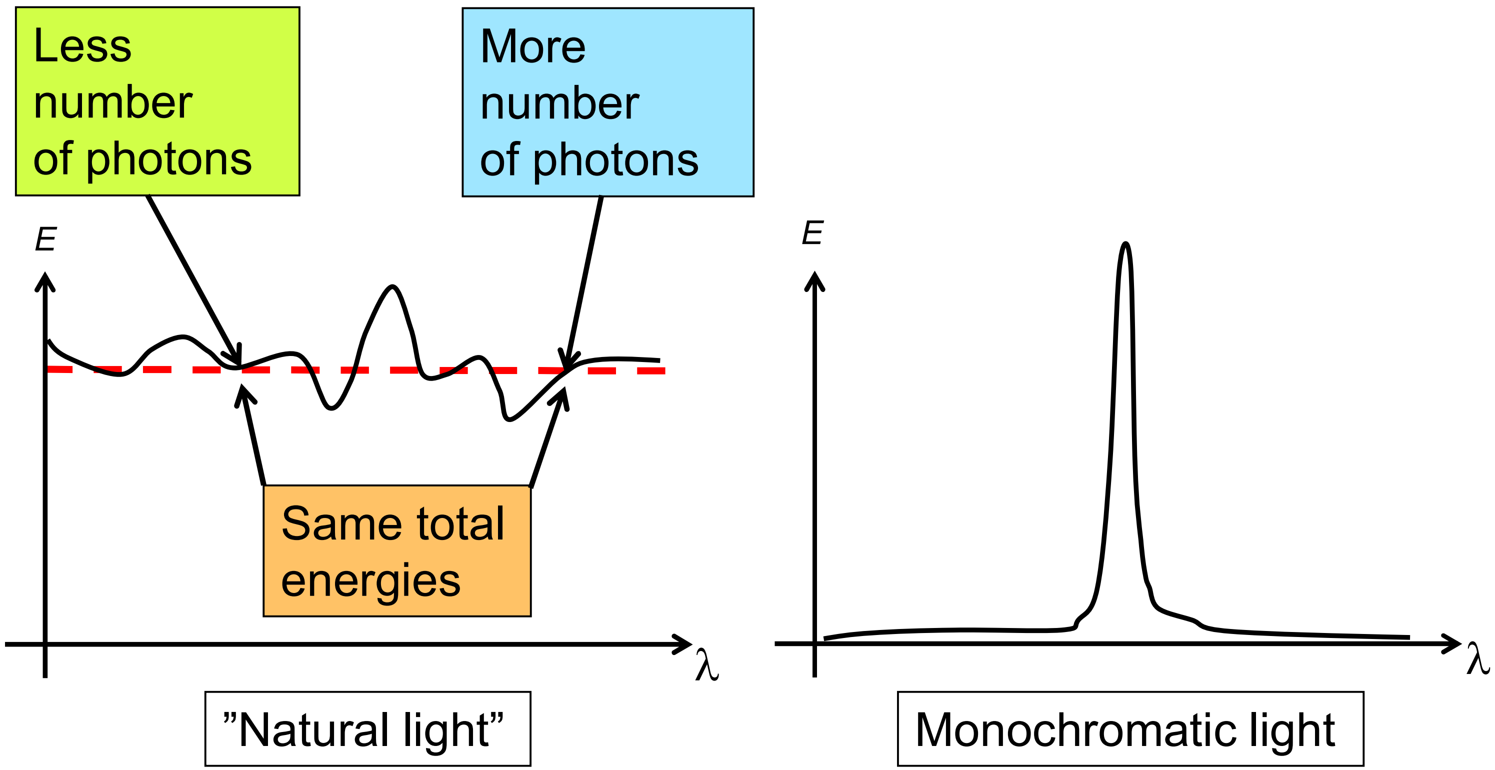
- If the speed of light changes from one medium to another,
  - the frequency ν is constant to make the energy constant
  - the wavelength λ must change

Spectrum

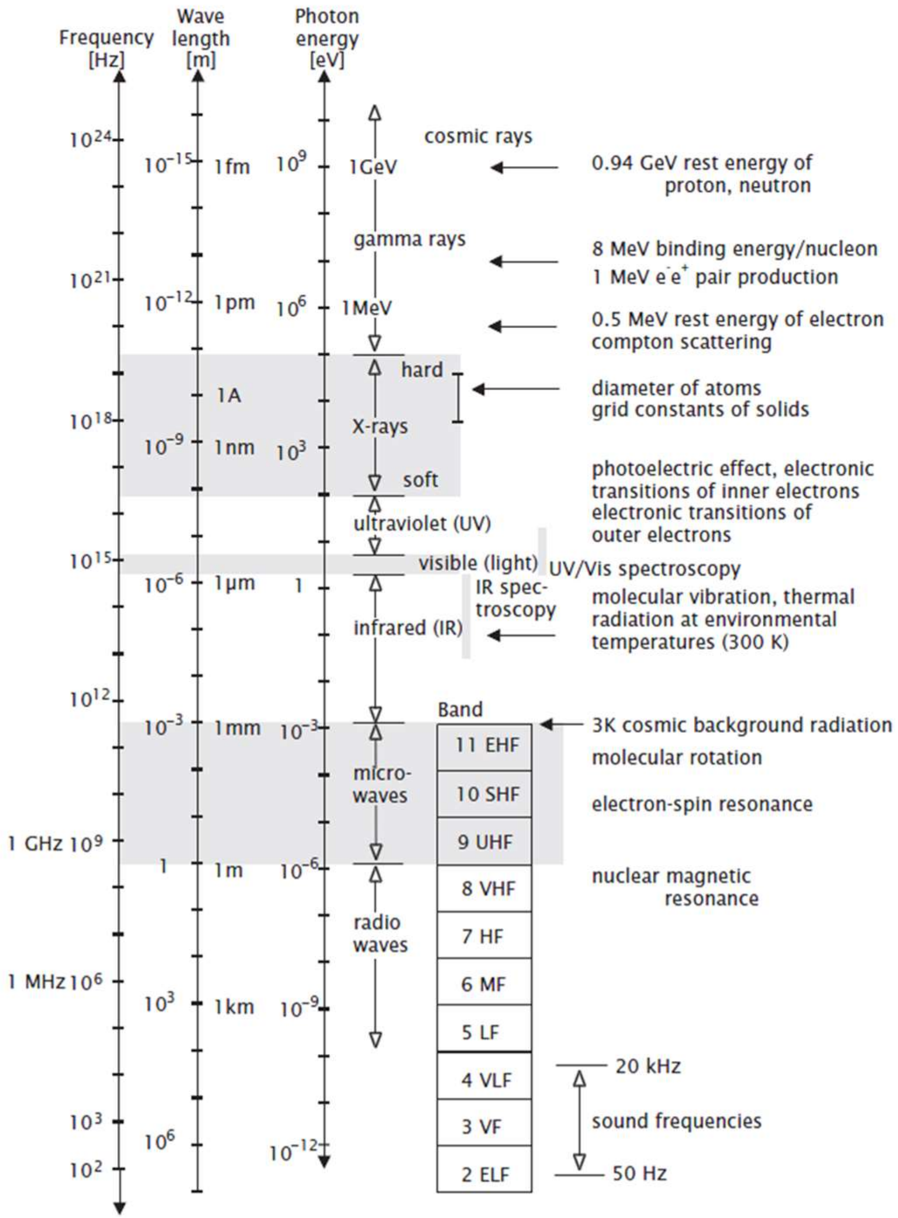
- In practice, light normally consists of
  - photons with a range of energies, or
  - waves with a range of frequencies
  - This mix of frequencies/wavelengths/energies is called the *spectrum* of the light
- The spectrum is a function that gives the *total amount* of energy for each frequency or wavelength
- Monochromatic light consists essentially of only one frequency/wavelength
  - Can be produced by special light sources, e.g., lasers

Spectrum

$$E = h\,\nu = h\,c\,/\,\lambda$$



Classifi-cation of the Electro-magnetic spectrum

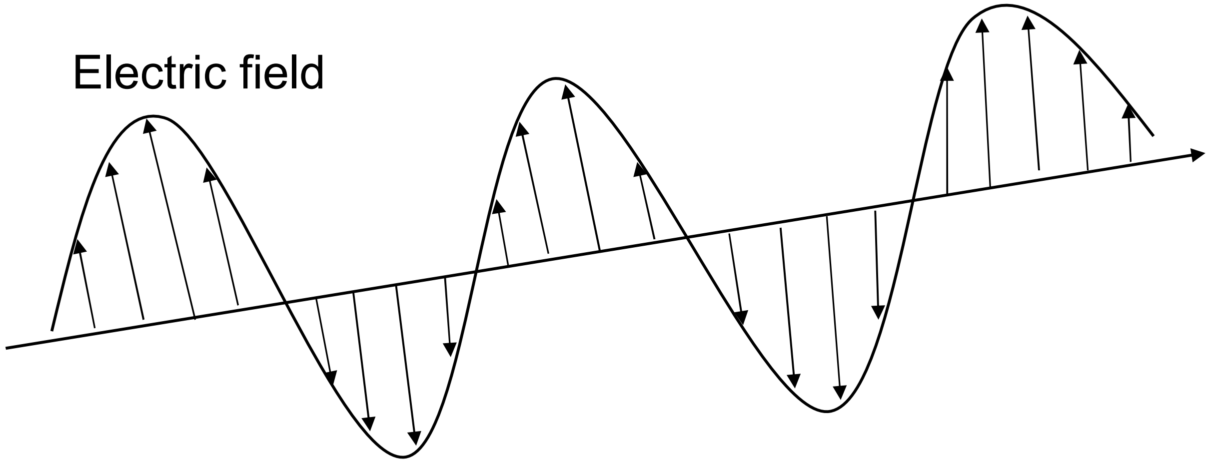


Polarization

- The electromagnetic field has a *direction*
  - Perpendicular to the direction of motion
- The *polarization* of the light is defined as the direction of the electric field
- Natural light is a mix of waves with polarization in all possible directions: *unpolarized light*
- Special light sources or filters can produce *polarized light* of well-defined polarization

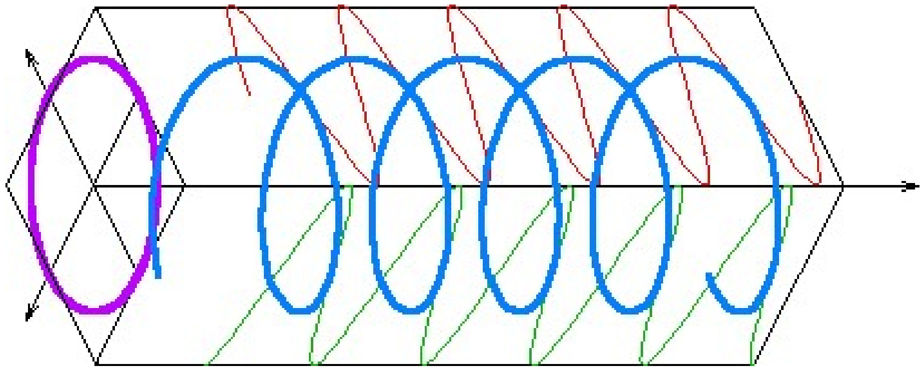
Polarization

- Plane polarization
  - The electric field varies only in a single plane



Polarization

- Circular/elliptical polarization
  - The electric field vector rotates
  - Can be constructed as the sum of two plane polarized waves with 90^{o} phase shift



- Conversely: plane polarized light can be decomposed as a sum of two circular polarized waves that rotate in opposite directions

Coherence

- The phase of the light waves can either be
  - random: *incoherent light* (natural light)
  - in a systematic relation: *coherent light*
- Coherent light is usually related to monochromatic light sources (e.g. laser)
- Compare a red light-emitting diode (LED) and a red laser
  - Both produce light within a narrow wave-length range
  - The red LED light is incoherent
  - The red laser light is coherent

Radiometry

- Light radiation has *energy*
  - Each photon has a particular energy related to its frequency ($E = h \nu$)
  - The number of photons of a particular frequency gives the amount of energy for this frequency
  - This is described by the spectrum
  - Unit: Joule (or Watt second)
  - Is usually not measured directly

Radiometry

- The power of the radiation, i.e., the energy per unit time, is the *radiant flux*
  - Since the energy depends on the frequency, so does the radiant flux
  - Unit: Watt or Joule per second
  - Is usually not measured directly

Radiometry

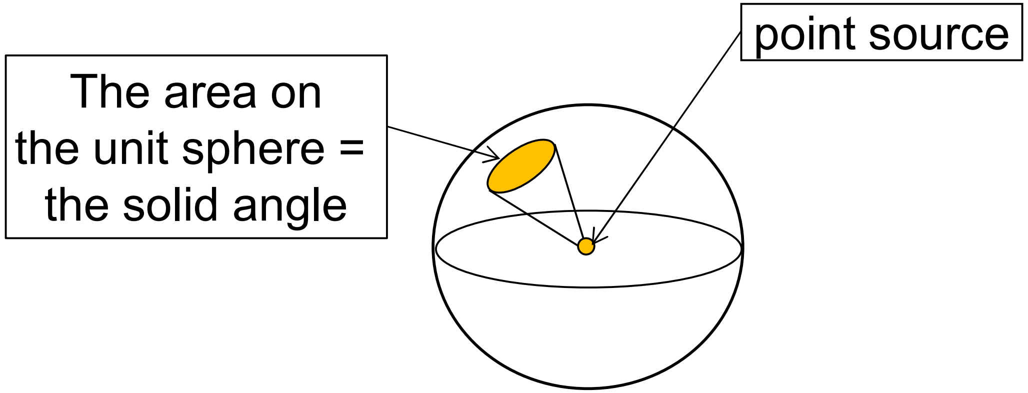
- The radiant flux per unit area is the *radiant flux density*
  - Since the flux depends on the frequency, so does the flux density
  - Unit: Watt per square meter
  - Can be measured directly!

As the energy through a specific area during a specific time interval

- *Irradiance*: flux density incident upon a surface
- *Excitance* or *emittance*: flux density emitted from a surface

Radiometry: Solid angle

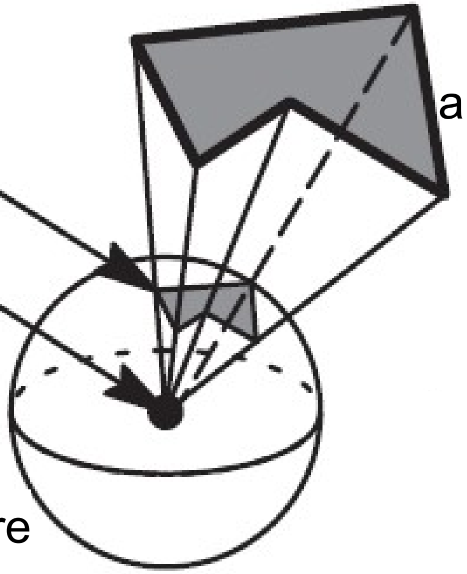
- For point sources or distant sources, the flux density can also be measured per unit solid angle



- The *radiant intensity* is the radiant flux per unit solid angle
  - Unit: Watt per steradian
    (the whole sphere corresponds to 4π steradians)

Radiometry: Solid angle

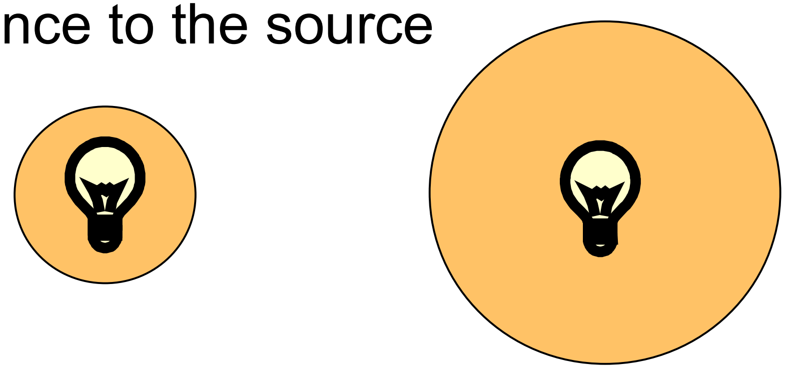
- The radiative energy decreases proportional to r^2



$$\text{Maximal solid angle} = \frac{A}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

Radiometry: Basic principle

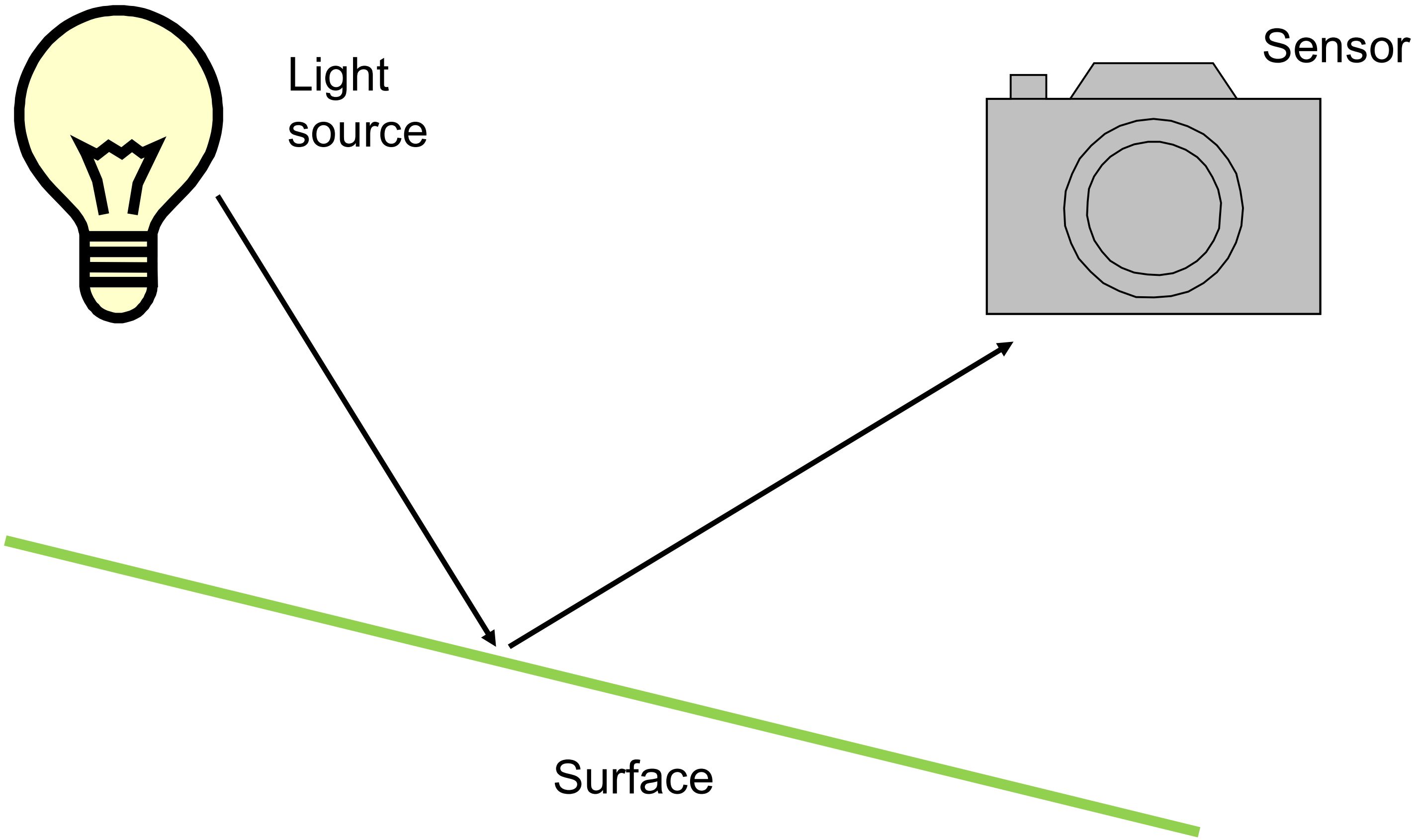
- Based on the preservation of energy
- A constant light source must produce the same amount of energy through a solid angle regardless of the distance to the source
  - The *radiant intensity* is constant with respect to distance
  - The *radiant flux* density decreases with the square of the distance to the source



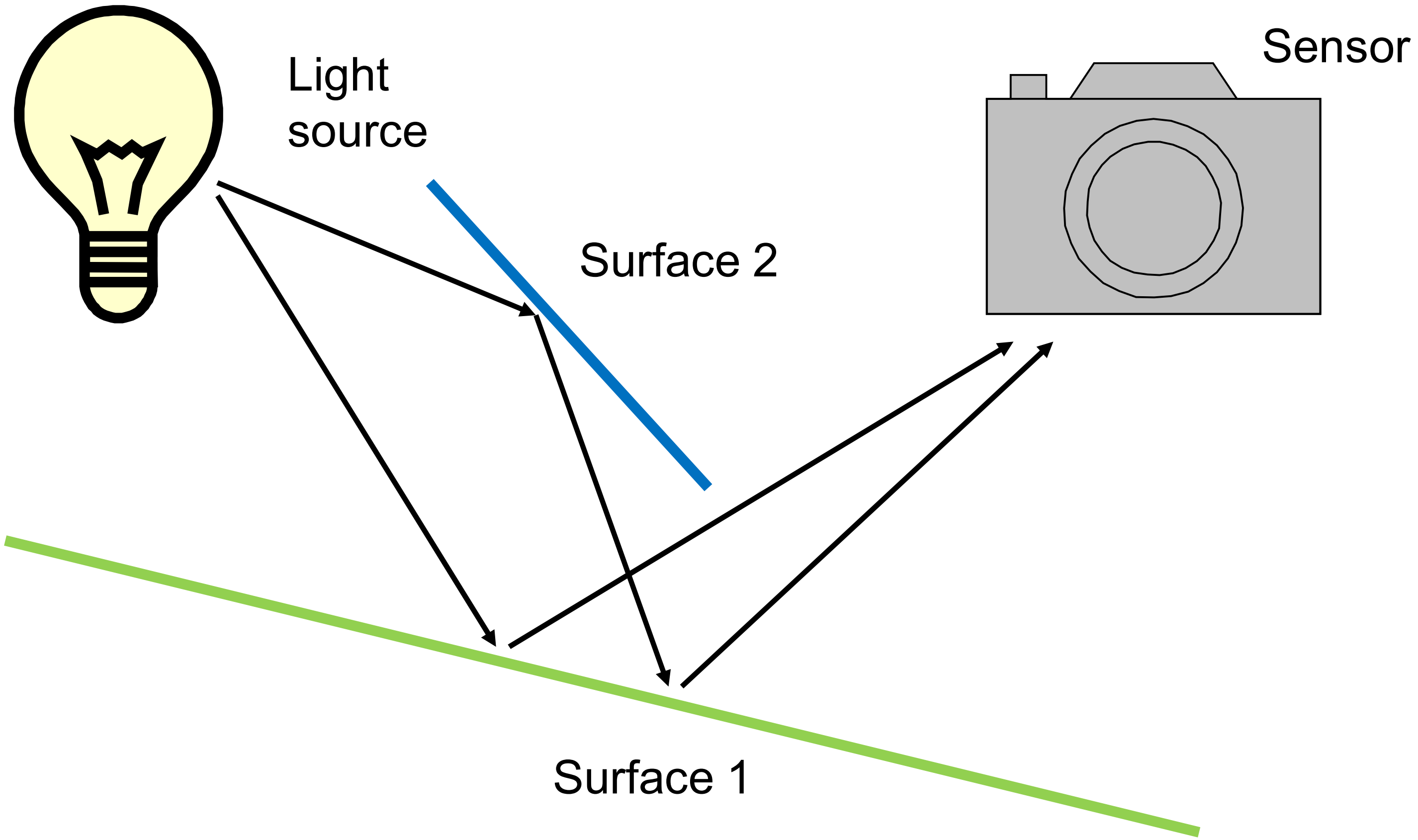
The radiometric chain

- By the radiometric chain; we mean the system consisting of:
  - light sources
  - objects that reflect the light
  - sensors that detect the light
  - medias which the light has to pass

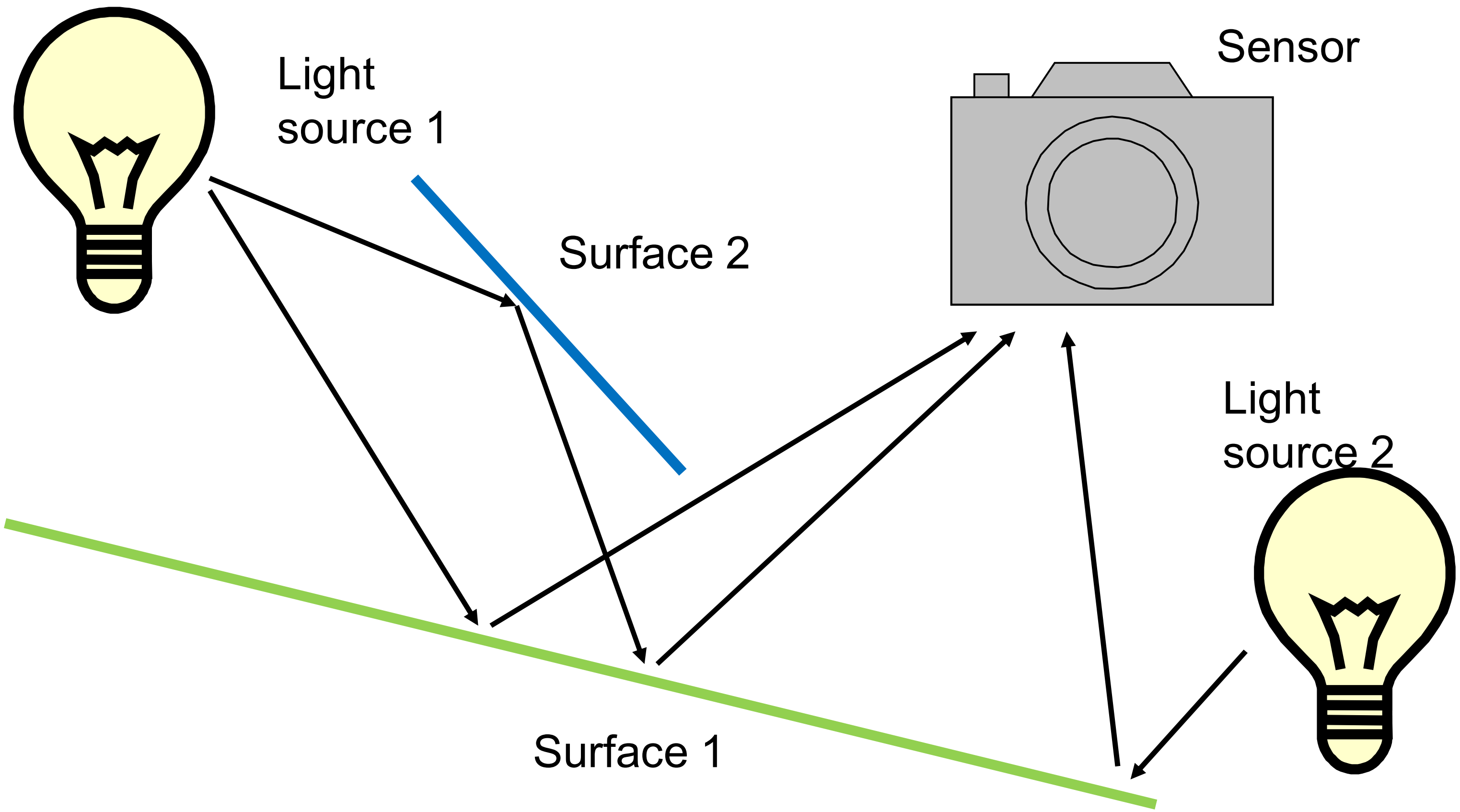
The radiometric chain



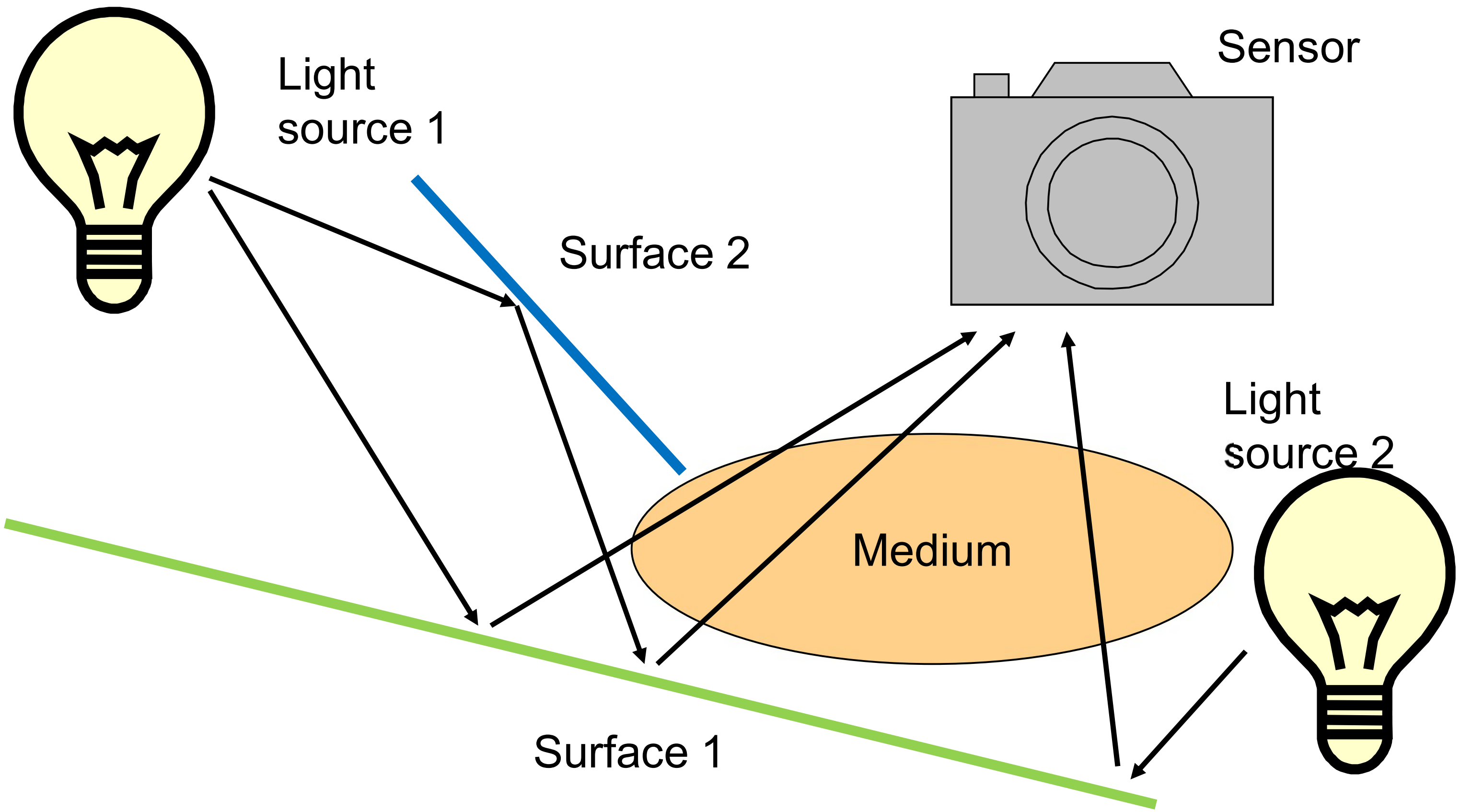
The radiometric chain



The radiometric chain



The radiometric chain



Interaction between light and matter

- Most types of light-matter interactions can be represented by
  - n = the material's refractive index
  - α = the material's absorption coefficient
- Both parameters depend on λ
- More complex interactions include polarization effects or non-linear effects

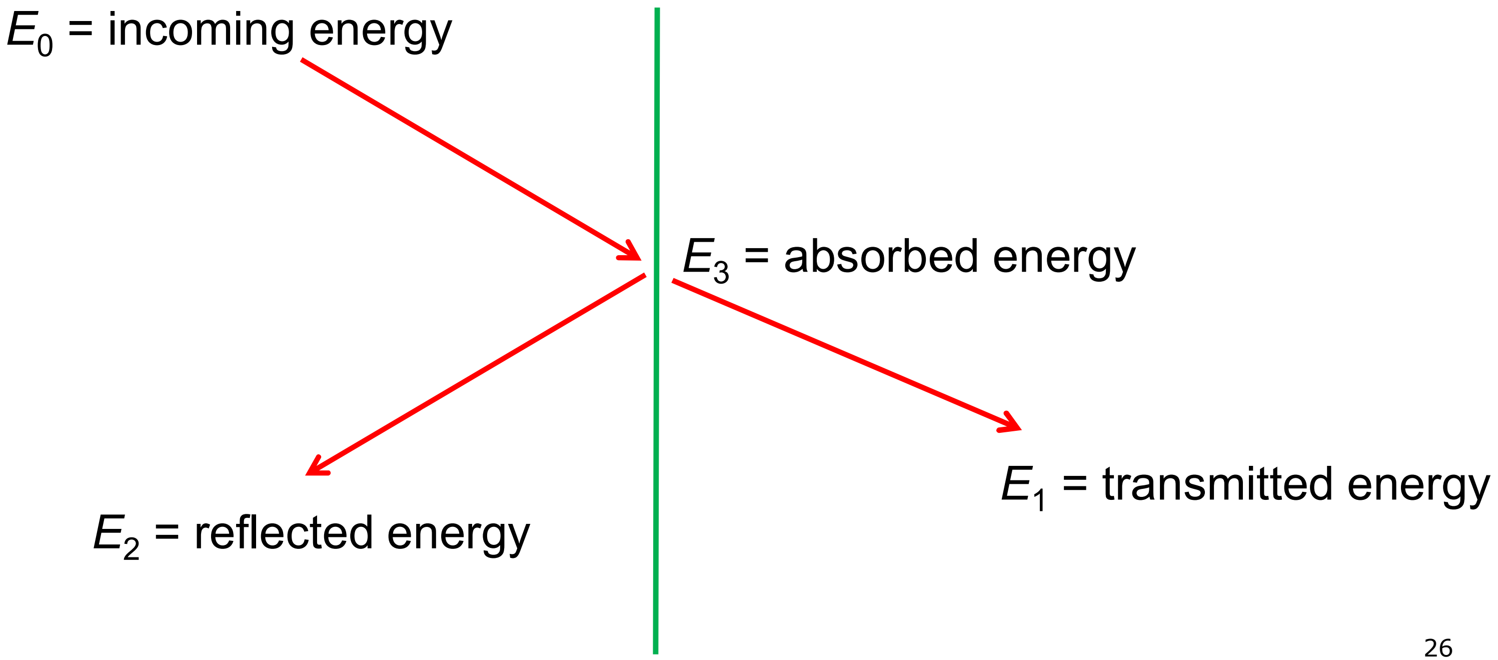
Light incident upon a surface

- When light meets a surface
  - Some part of it is transmitted through the new media
    - Possibly with another speed and direction
  - Some part of it is absorbed by the new media
    - Usually: the light energy is transformed to heat
  - Some part of it is reflected

- For the same material, all three effects depend on the light's wavelength λ
  - Equivalently: they depend on the light's frequency ν

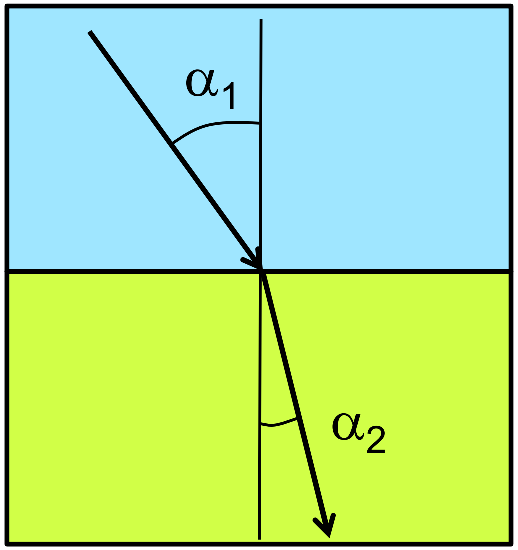
Basic principle

- Based on the preservation of energy:
  $E_0 = E_1 + E_2 + E_3$



Refraction

- The light that is transmitted into the new medium is *refracted* due to the change in light speed



Snell's law of refraction:

$$\frac{\sin\alpha_2}{\sin\alpha_1} = \frac{n_1}{n_2} = \frac{c_2}{c_1}$$

Absorption

- Absorption implies attenuation of transmitted or reflected light
- Materials get their colors as a result of different amounts of absorption for different wavelengths
  - Ex: A red object attenuates wavelengths in the red band less than in other bands. These wavelengths are reflected (or transmitted) instead.

Absorption

- The absorption of light in matter depends on the length that the light travels through the material:

$$a = e^{-\alpha x}$$

  - a = attenuation of the light ($0 \leq a \leq 1$)
  - α = the material's absorption coefficient
  - x = length that the light travels in the material

Absorption spectrum

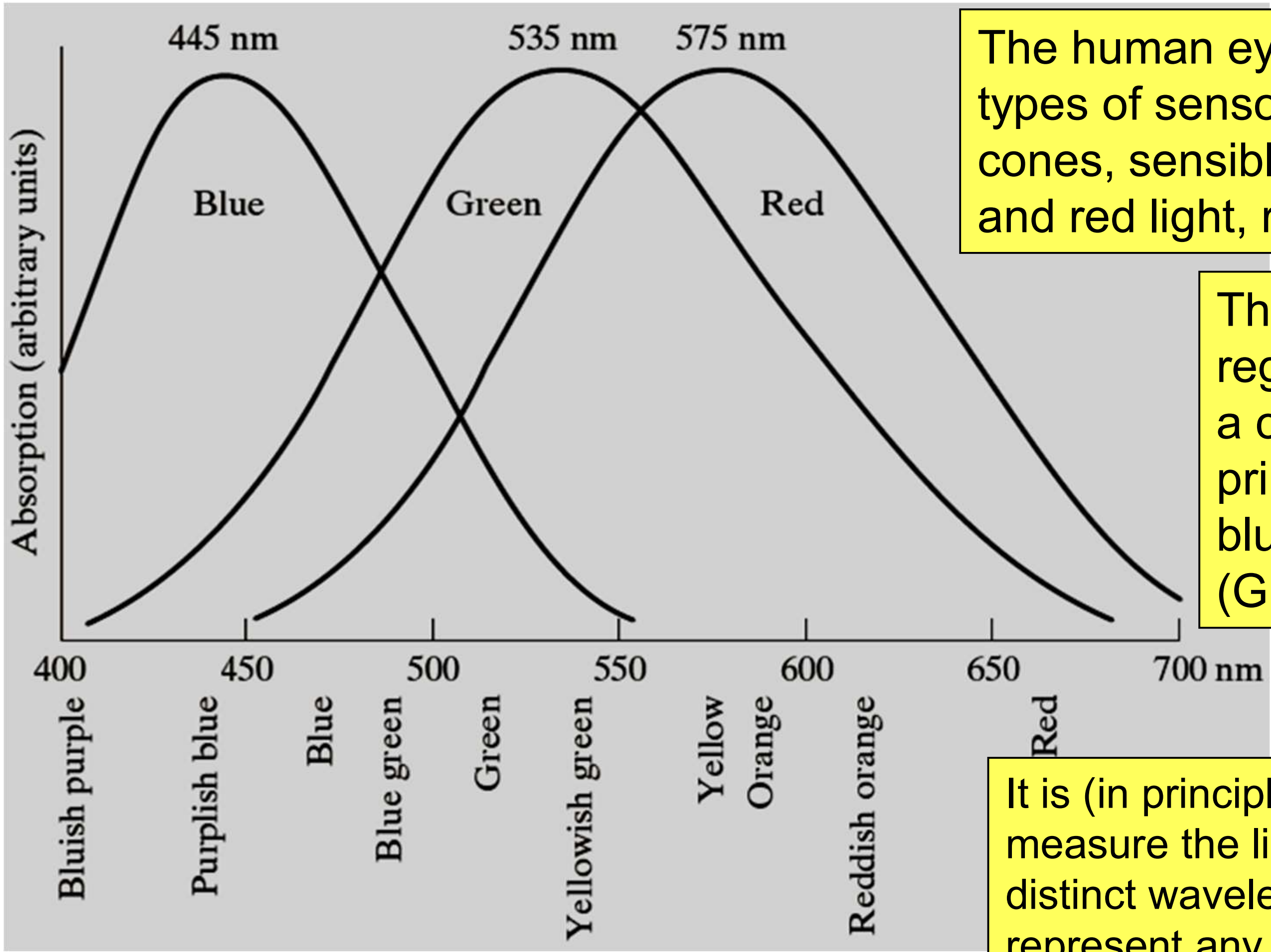
- The spectrum of the reflected/transmitted light is given by

$$s_2(\nu) = s_1(\nu)a(\nu)$$

  - s_1 = incident spectrum
  - s_2 = reflected/transmitted spectrum
  - a = absorption spectrum $(0 \le a(\nu) \le 1)$

Absorption of light by the cones in the human eye

Figure from: Gonzales & Woods p. 31



The human eye contains 3 types of sensors named cones, sensible to blue, green and red light, respectively.

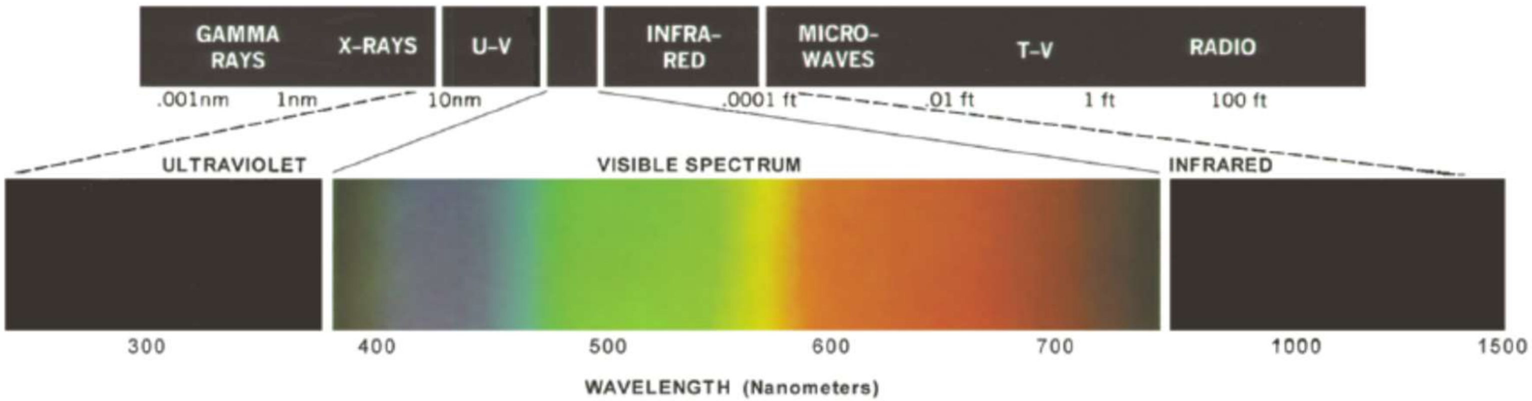
The human eye regards a color as a combination of 3 primary colors blue (B), green (G) and red (R).

It is (in principle) sufficient to measure the light spectrum in 3 distinct wavelength bands to represent any perceivable color.

Figure from:
Gonzales & Woods

The full color spectrum

- To perform correct calculations with color, the full color spectrum with individual wavelengths should be regarded.

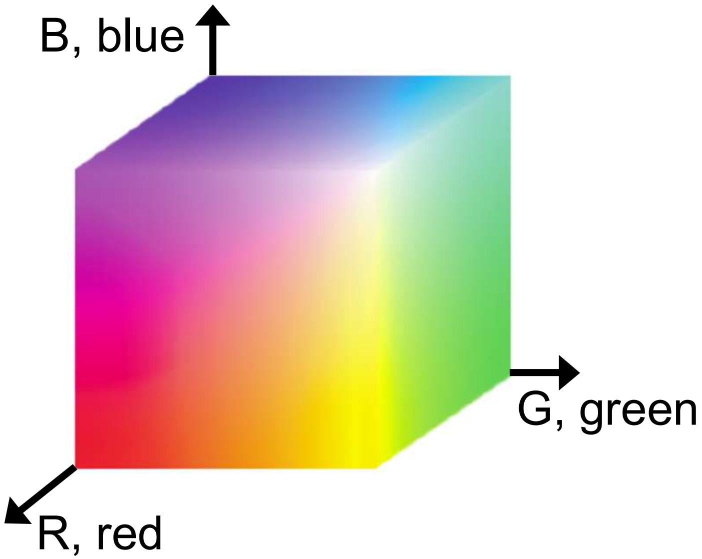


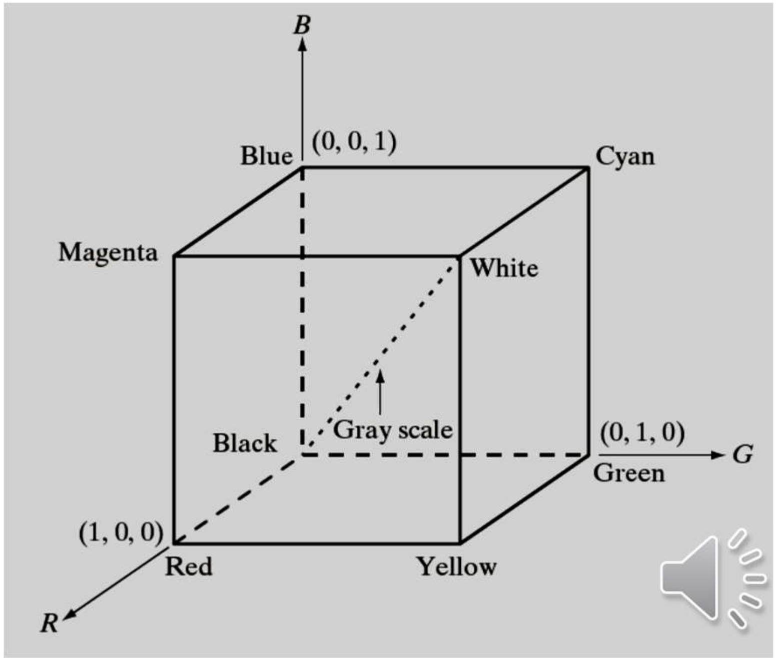
- The RGB color model is a simplified, approximate model that often works well, see next slide.

Figure from:
Gonzales & Woods

The simplified RGB color model

- [R G B] can be regarded as an orthogonal coordi-nate system. It assumes additive color mixing.
- [R G B] = [1 1 1] corresponds to white.
- [R G B] = [0 0 0] corresponds to black.



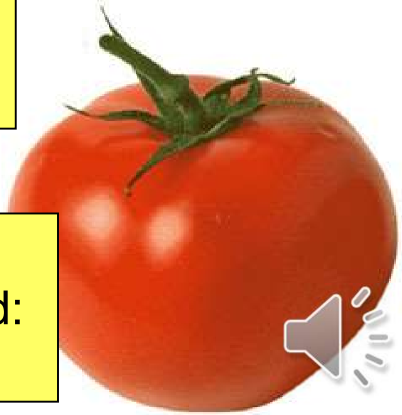


Absorption and color of objects

Assume white light: [1 1 1]

An object that reflects light in all wave lengths (all RGB) appears white:
[1 1 1]·[1 1 1]=[1 1 1]

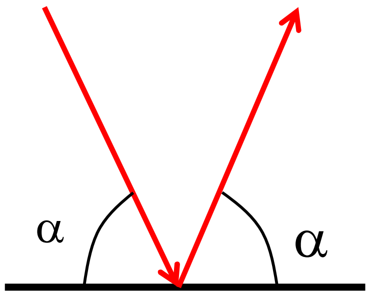
An object that reflects blue light and absorbs green-red light appears blue:
[1 1 1]·[0 0 1]=[0 0 1]



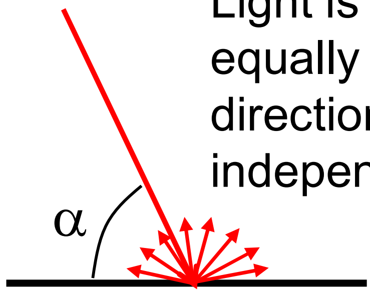
An object that reflects red light and absorbs blue-green light appears red:
[1 1 1]·[1 0 0]=[1 0 0]

Surface reflection

- The surface reflection is highly dependent on the surface type:



Mirror



Light is reflected equally much in all directions independent of α

Diffuse surface = Lambertian surface

- A real surface is often a mix between the two cases.

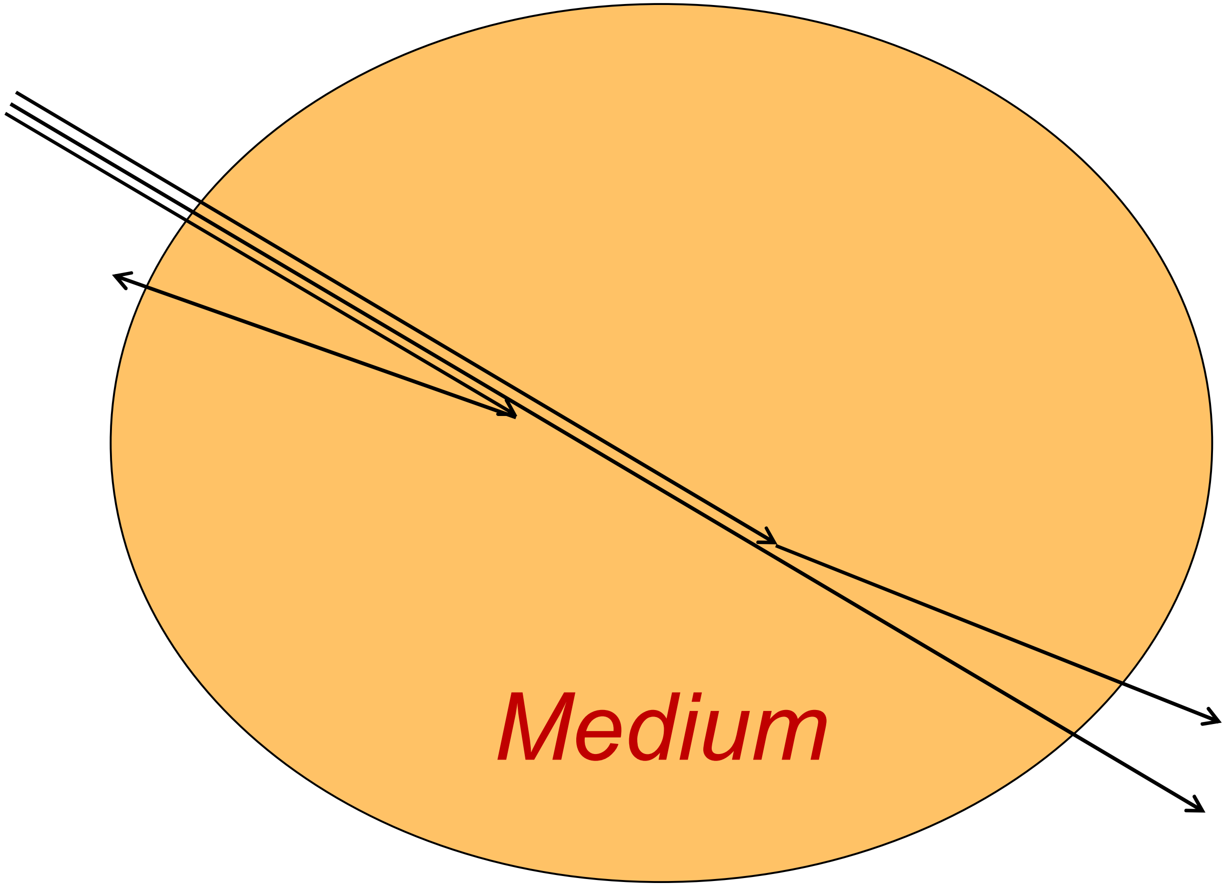
Emission

- Independent of its interaction with incident light (well, almost…):
  - Any object, even one that is not considered a light source, emits electromagnetic radiation
- Primarily in the IR-band, based on its temperature

Scattering

- All mediums (other than vacuum) *scatter* light
  - Examples: air, water, glass
- We can think of the medium as consisting of small particles and with some probability they reflect the light
  - In any possible direction
  - Different probability for different directions
  - Weak effect and roughly proportional to λ^{-4}
  - In general, the probability depends also on the distribution of particle sizes

Scattering

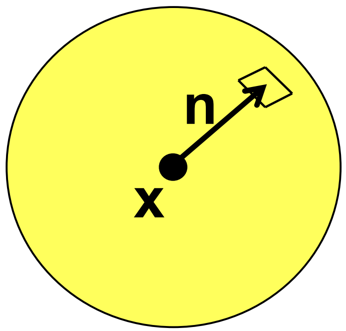


Scattering

- Scattering is not an absorption
- It rather means that the light ray does not travel along a straight line through the medium
  - There is a probability that a certain photon exits the medium in another direction than it entered.
- Examples:
  - The sky is blue because of the scattering of the sun light
    - Blue light has shorter wavelength and is scattered more by particles in the atmosphere
  - A strong laser beam becomes visible in air

The plenoptic function

- At a point $\mathbf{x} = (x_1,x_2,x_3)$ in space we can measure how much light energy that travels in the direction $\mathbf{n} = (n_1,n_2,n_3)$,
  $||\mathbf{n}|| = 1$



The plenoptic function

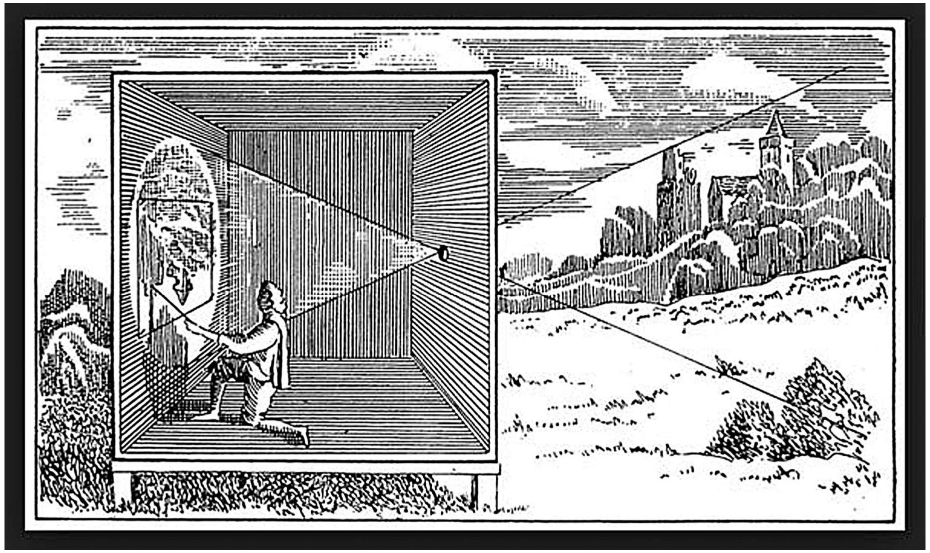
- The plenoptic function is the corresponding radiance intensity function
  - $p(\mathbf{x},\mathbf{n})$ (5-dim. since **x** is 3-dim. and **n** has 2 degrees of freedom.)
- Can also be a function of
  - Frequency ν
  - Time t
  - $p(\mathbf{x},\mathbf{n},\nu,t)$ (7-dim)
  - (Polarization)

A light camera

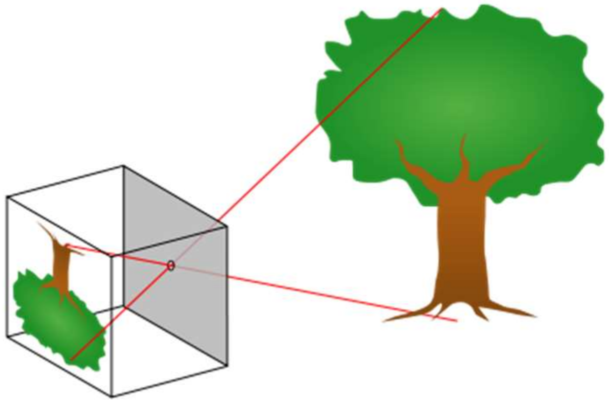
- A (light) camera is a device that *samples* the plenoptic function in a particular way
- Different types of cameras sample in different ways
  - Pinhole-camera
  - Orthographic camera
  - Push-broom camera
  - Light-field camera
  - …

The pinhole camera

- The most common camera model is the *pinhole camera*
  - Swedish: *hålkamera*
- An ideal model of the *camera obscura*



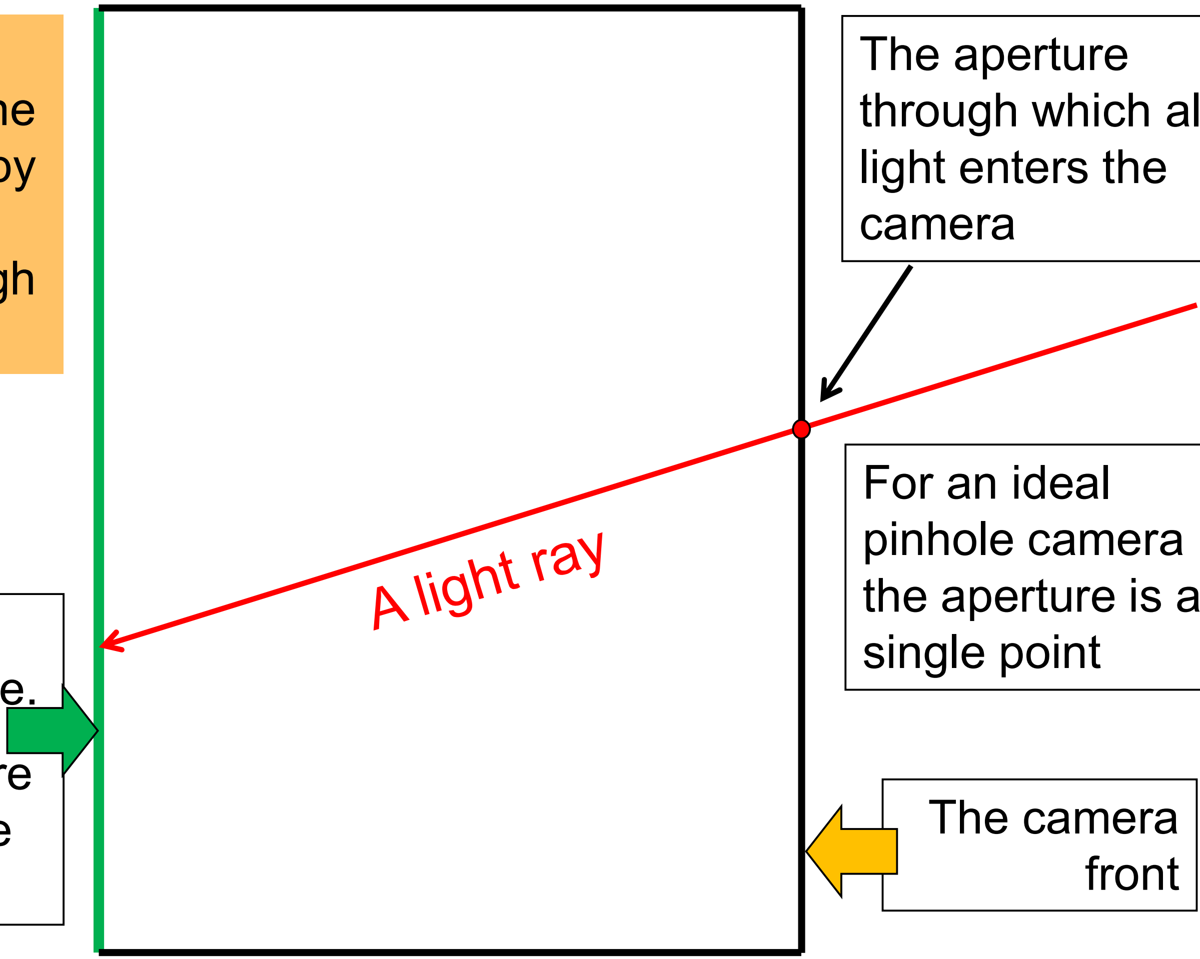
Athanasius Kircher, Large portable camera obscura, 1646.



The pinhole camera model

Each point in the image plane is illuminated by a single ray passing through the aperture

The aperture through which all light enters the camera



For an ideal pinhole camera the aperture is a single point

The image plane is here.

This is where we measure the image.

The camera front

The pinhole camera model

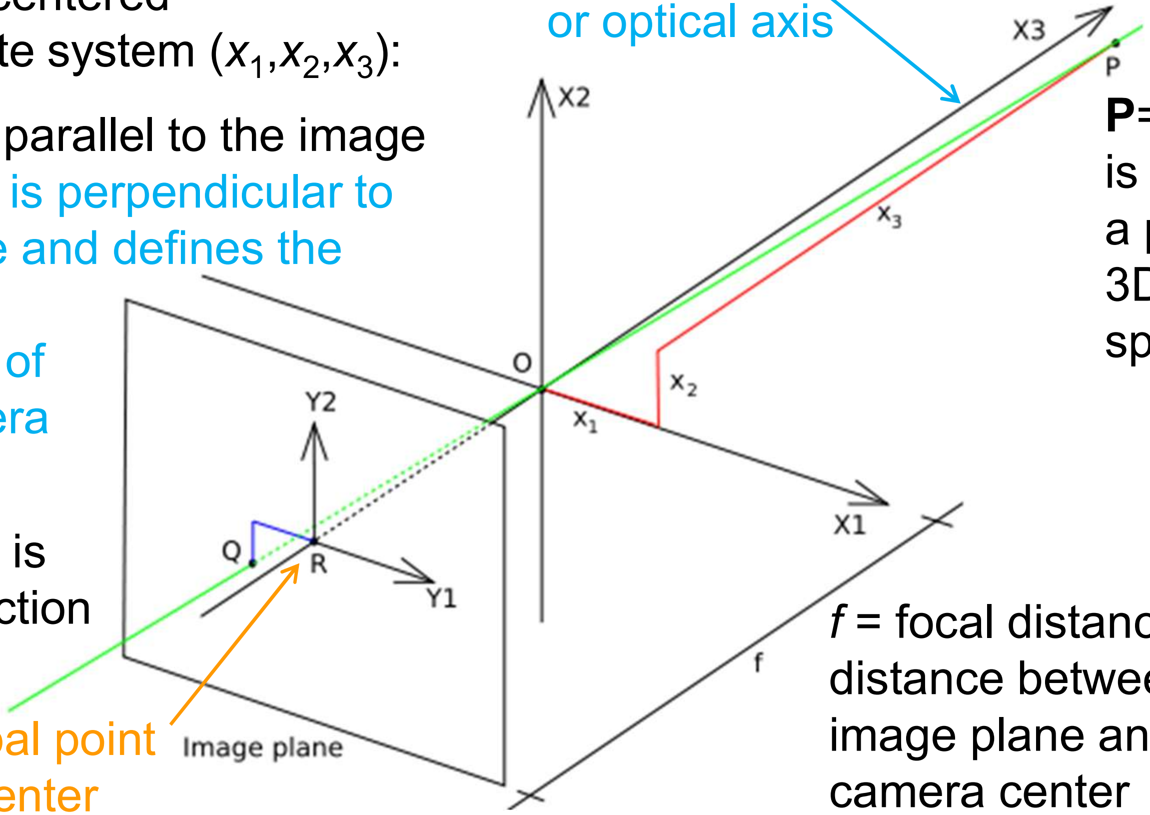
- Mathematically we need only know the location of the image plane and the aperture
  - The rest is physics + practical implementation
- In the literature, the aperture point is also called
  - camera center
  - camera focal point

The pinhole camera model

The image plane and the camera center define a camera-centered coordinate system (x_1,x_2,x_3):

x_1,x_2 are parallel to the image plane, x_3 is perpendicular to the plane and defines the viewing direction of the camera

The principal or optical axis



P$=(x_1,x_2,x_3)$ is a point in 3D space

Q$=(y_1,y_2)$ is the projection of **P**

The principal point or image center

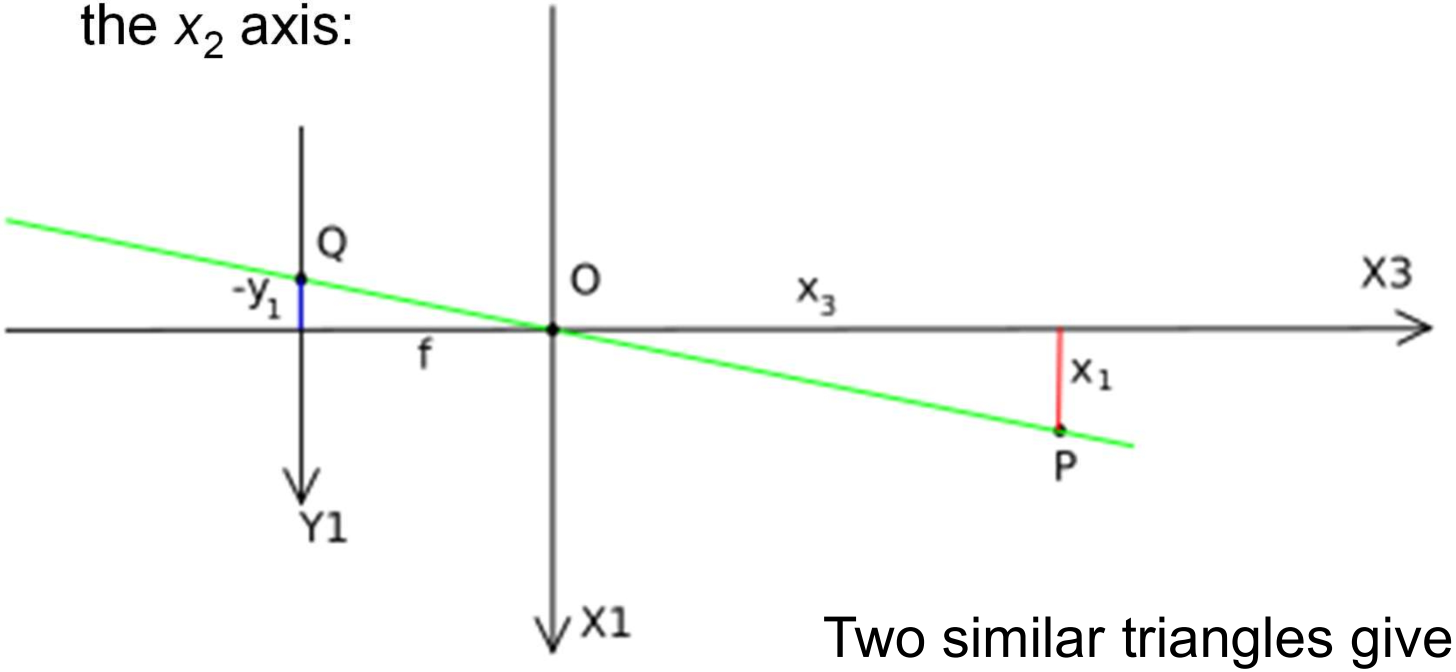
f = focal distance, the distance between the image plane and the camera center

The pinhole camera model

- **R** is the point where the optical axis intersects the image plane
  - The *principal point* or the *image center*
- The (x_1, x_2) plane is the *principal plane* or *focal plane*
- The green line is the *projection line* of point **P**
  - All points on the line are projected onto **Q**
  - Alternatively: the projection line of **Q**

The pinhole camera model

- Let us look at the camera coordinate system along the x_2 axis:



Two similar triangles give:

$$\boxed{\frac{-y_1}{f} = \frac{x_1}{x_3} \quad \text{or} \quad y_1 = -\frac{f\ x_1}{x_3}}$$

The pinhole camera model

- Looking along the x_1 axis gives a similar expression for y_2
- This can be summarized as:

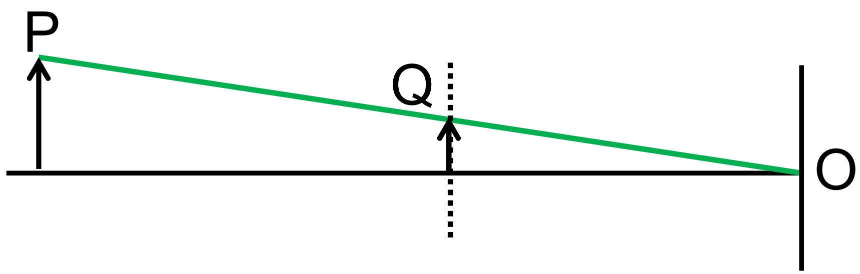
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = -\frac{f}{x_3} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The virtual image plane

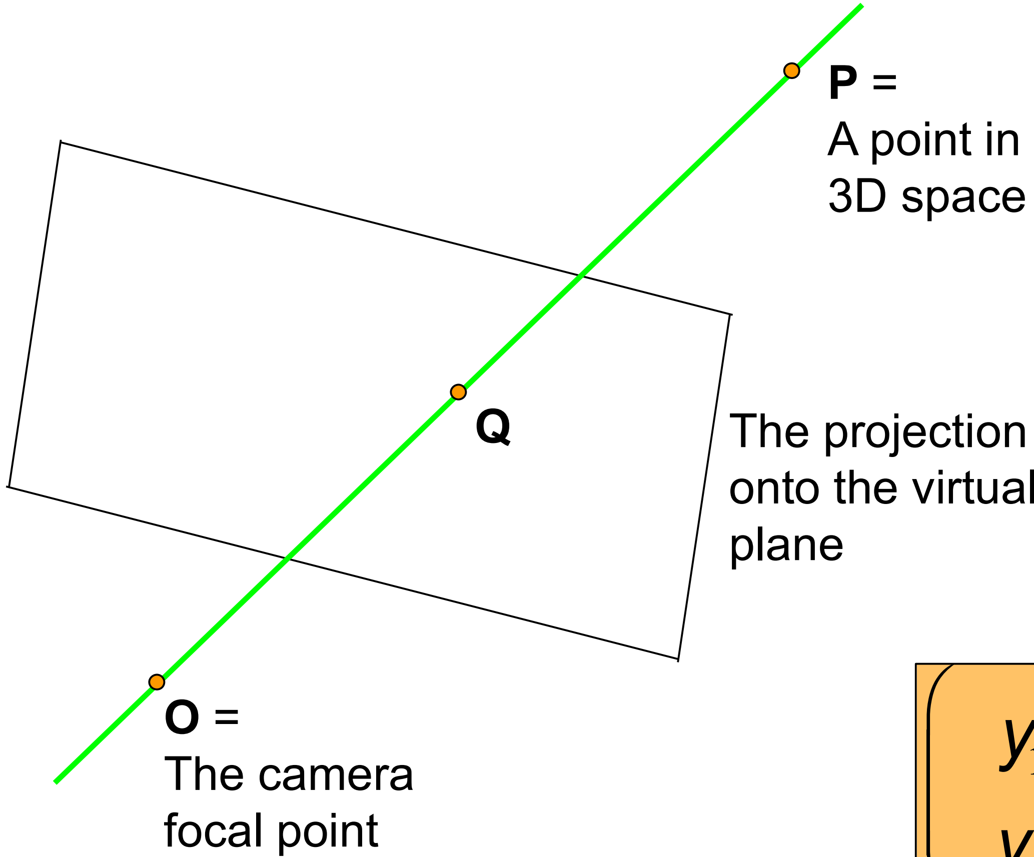
- The projected image is rotated 180^{o} relative to how we “see” the 3D world
  - Reflection in both y_1 and y_2 coordinates = rotation
- Must be de-rotated before we can view it
  - In the film-based camera, the image is manually rotated
  - In the digital camera this is taken care of by reading out the pixels in the “rotated” order
  - For us humans, with the retina as image plane, the brain performs the rotation
- Mathematically this is equivalent to placing the image plane in front of the focal point

The virtual image plane

- Projection lines works as before: from **P** through the focal point and intersect at **Q**
- This defines the *virtual image plane*
  - Cannot be realized in practice
  - Produces the same image as the rotated image from the real image plane



The virtual image plane = a perspective projection



$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \frac{f}{x_3} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

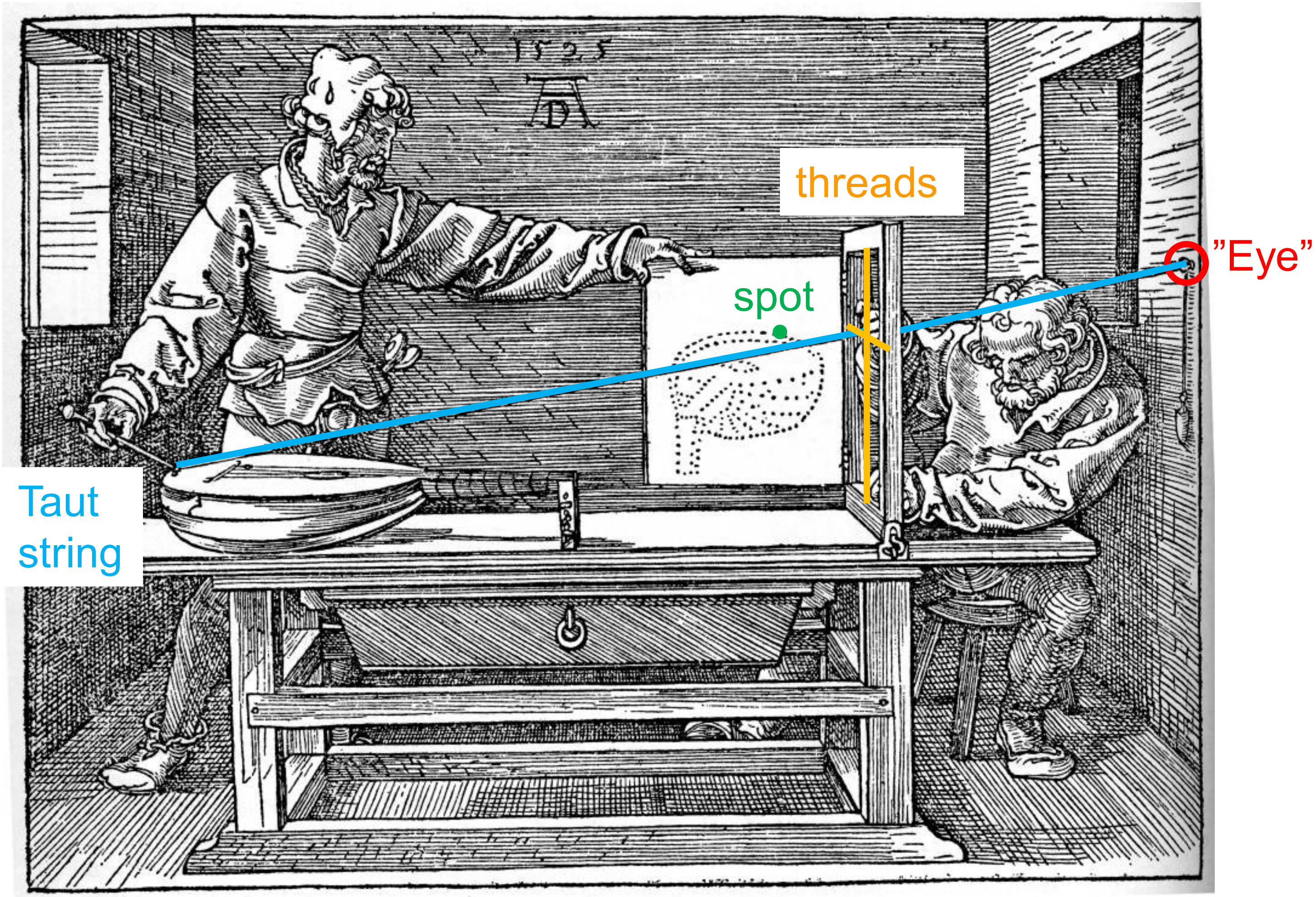
The perspective projection

- Albrecht Dürer, painter, self portrait
  - born 1471 in Nürnberg, died 1528 in Nürnberg
  - You can visit his house in Nürnberg!

The perspective projection 1

- See next slide.
- Dürer published a book about the art of measurement in 1525 shortly before his death in 1528 and this illustration of a method of perspective drawing was the final image in the book.
- In this wonderful device for painstakingly constructing a correct perspective, the imaginary eye is physically represented by the ring in the wall, and the point where each part of the subject appears in the picture plane is measured from the taut string and then transferred to the picture as a dot.
- Two movable crosswise threads are attached to the frame. They are used to mark the position of the long thread.
- The door of the frame can be closed. Mark the spot where the threads cross the tablet.
- Continue with another point…

The perspective projection 1

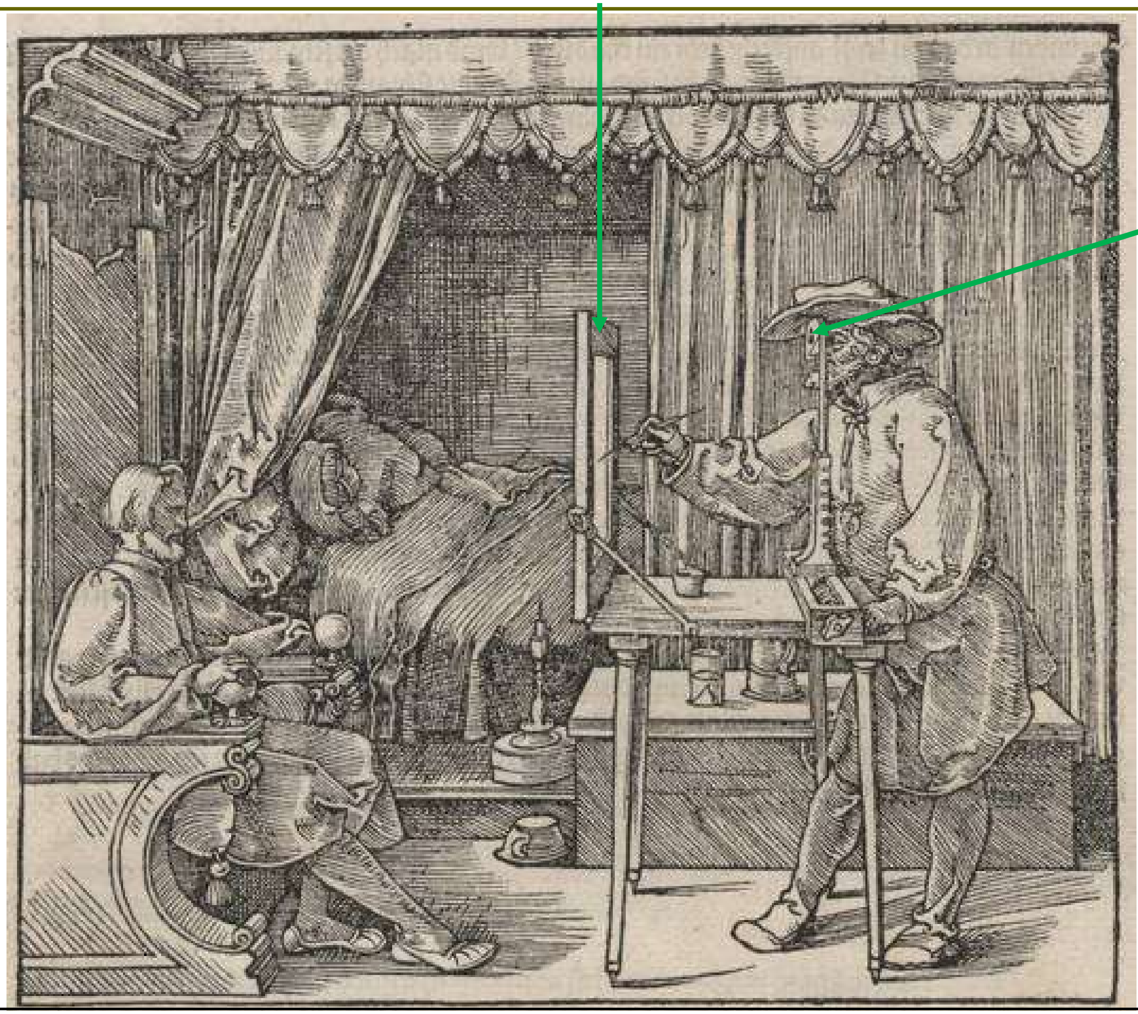


The perspective projection 2

- See next slide.
- A more practical method for actually producing a perspective drawing is shown here. Dürer rather caustically remarks: “Such is good for all those who want to make a portrait and who are not confident of their skill.”

The perspective projection 2

Transparent paper



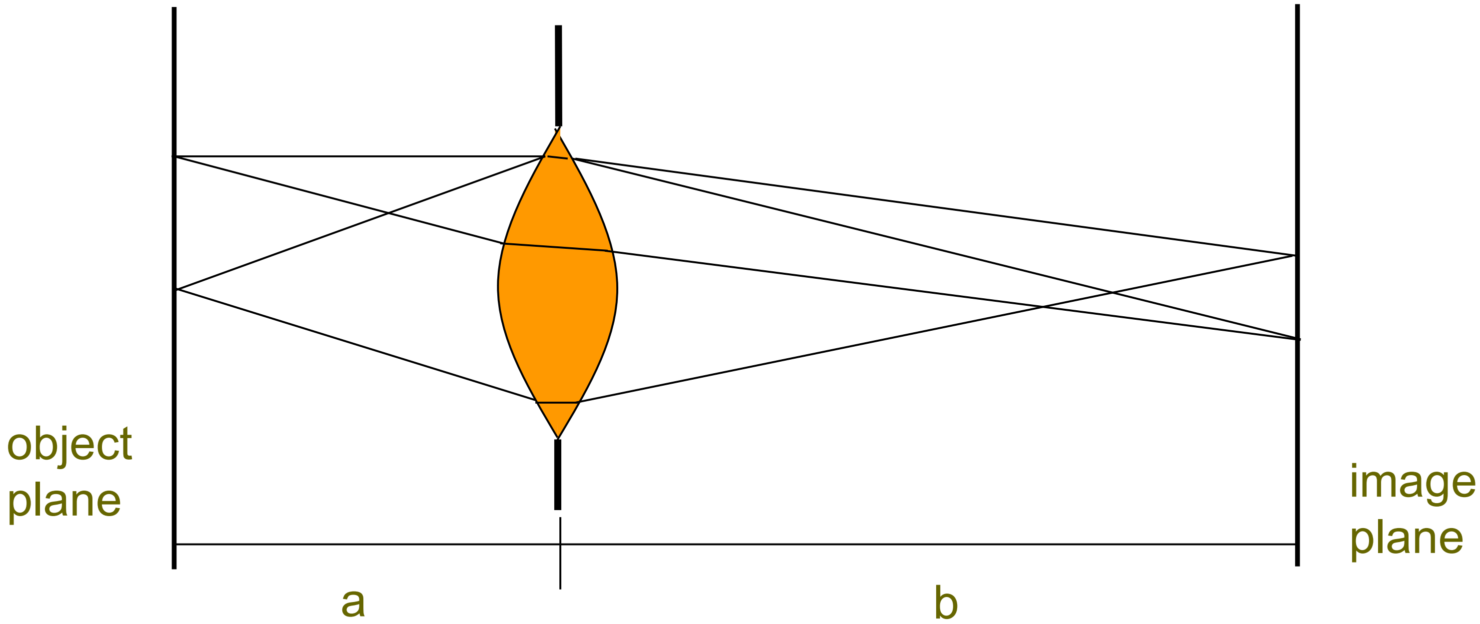
Hole to look through

Lenses vs. infinitesimal aperture

- The pinhole camera model doesn’t work particularly well in practice since
  - If we make the aperture small, too little light enters the camera
  - If we make the aperture larger, the image becomes blurred
- Solution: we replace the aperture with a lens or a system of lenses
- Disadvantage:
  - The lens camera only gives a perfect sharp image for objects in the object plane, see slides ahead.

Thin lenses

- The simplest model of a lens
- Focuses all points in an *object plane* onto the image plane



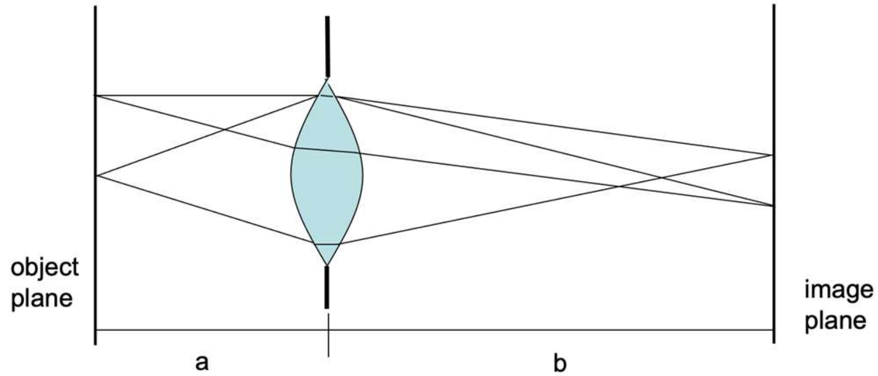
The object plane

- The object plane consists of all points that appear sharp when projected through the lens onto the image plane
- The object plane is an ideal model of where the “sharp points” are located
  - In practice: the object plane may be non-planar: e.g. described by the surface of a sphere
  - The shape of the object plane depends on the quality of the lens (or lens system)
  - For thin lenses the object plane can often be approximated as a plane

Thin lenses

- The thin lens is characterized by a single parameter: the *focal length* f_L

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{f_L}$$



- To change *a* (distance to object plane), we need to change *b* since f_L is constant
- $a = \infty$ for $b = f_L$!

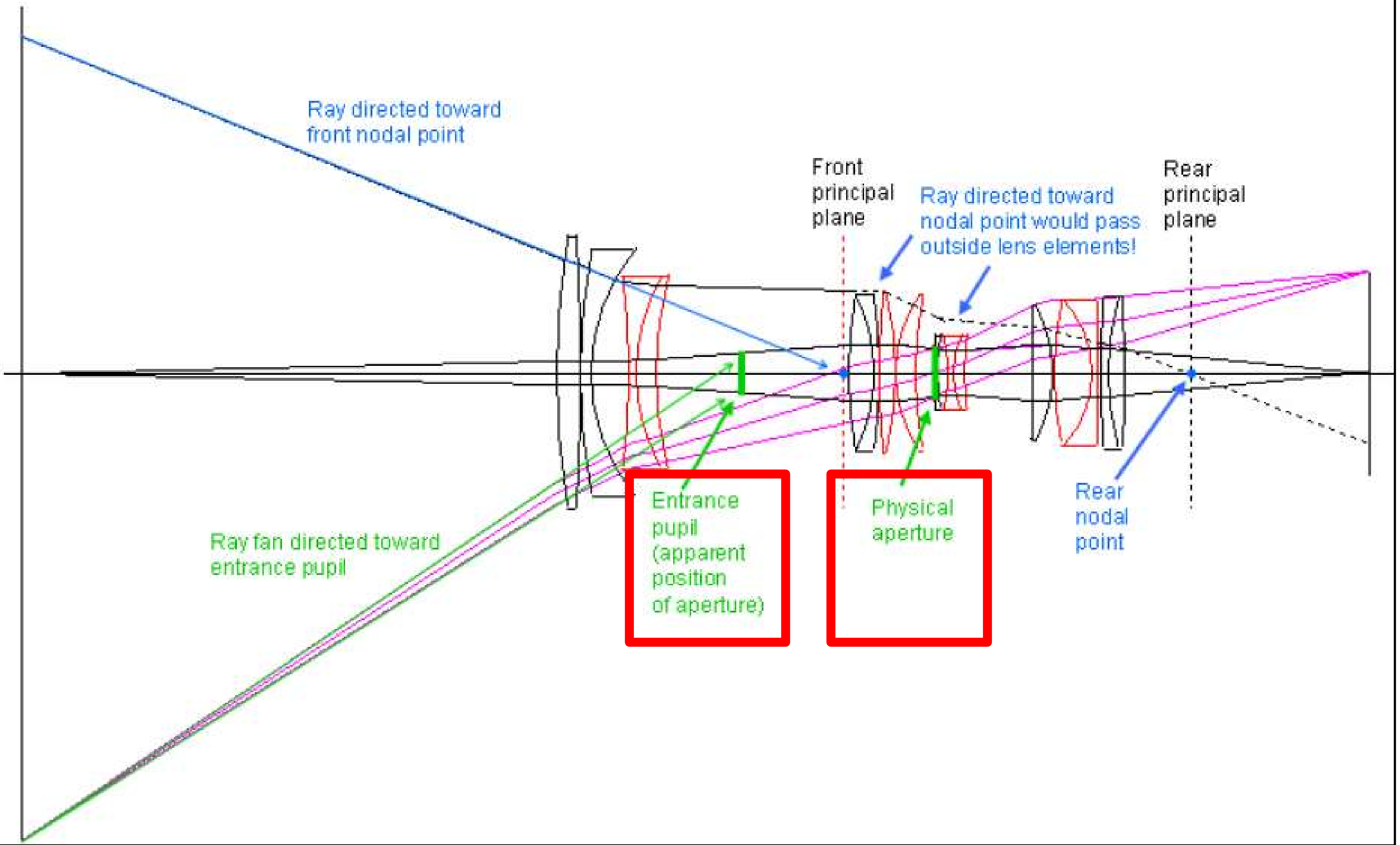
Where is the camera center in a real lens?

- The camera center is at EP (the entrance pupil) i.e. the apparent position of the aperture.

Entrance Pupil for the Pentax Super-Takumar 200mm f/4 lens

Reference: Theory of the "No-Parallax" Point in Panorama Photography
Version 1.0, February 6, 2006
Rik Littlefield (rj.littlefield@computer.org)

Where is the camera center in a real lens?



End of Lecture 1 Image Formation. Questions?

Topics we will return to and deepen:

- Emission, Absorption
  - Lectures: “The IR sensor”, “3D visualization”
- Surface reflection
  - Lecture: “3D visualization”
- The plenoptic function and special light cameras
  - Lecture: “Specialized cameras”
- The pinhole camera & The virtual image plane
  - Lectures: “Camera calibration 1 & 2”
- Lenses
  - Lecture 5 Image Formation, lens issues.