

# Guide to answers for written examination in TSBB09 Image Sensors, 2016-01-11

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## PART I: STANDARD CAMERAS & IR SENSORS

**Exercise 1** See lecture A, slides 30-33.

**Exercise 2** See lecture A, slides 88-89.

**Exercise 3** IR-sensors with active cooling can have higher frame rate (image per second) than uncooled sensors, and also have higher sensitivity. See lecture C, page 6.

**Exercise 4** CYM filters block less light than RGB filters, therefore they have a higher sensitivity (more photons are converted to electrons). See lecture B, slide 65.

**Exercise 5 Pinhole camera.** Advantage: All 3D points projected onto the image are sharp. Disadvantage: all light must pass through one small point, so only little light enters the camera which requires relatively long exposure time.  
**Lens based camera.** Advantage: light enters through a larger opening, allows shorter exposure time. Disadvantage: only 3D points in, or close to, the object plane (defined by the geometry of the lenses) are projected sharply in the image. See lecture A, slide 60.

**Exercise 6** A positive potential on the upper part of the transistor generates an electric field that forces the electrons to stay under the transistor. See lecture B, slides 21-22.

**Exercise 7** The digital value  $D$  corresponding to a specific pixel is (approximately) proportional to the intensity of light  $I$  that falls onto the pixel:  $D = g \cdot I$ . The gain  $g$  is dependent, for example, on the pixel area, the exposure time, the quantum efficiency, and on the analog-to-digital conversion.

**Exercise 8** The two curves show how much energy a surface radiates (vertical axis) in different wavelengths (horizontal axis), for two different temperature. The “higher” curve, which peaks at a shorter wavelength, is for the higher temperature. The colored areas indicate the wavelength band where a particular IR-sensor has its optimal sensitivity, i.e., the output for a particular pixel is proportional to the colored area and therefore depends on the temperature of the surface that projects onto this pixel.

## PART II: GEOMETRY AND MULTIPLE VIEWS

**Exercise 9** See lecture E, slide 15.  $\mathbf{y}_1$  and  $\mathbf{y}_2$  are the homogeneous coordinates of corresponding image points.

**Exercise 10** The two optical axes become parallel and perpendicular to the baseline (= the line between the two camera centers).

**Exercise 11** The different images should be taken with cameras with (approximately) the same camera center.

**Exercise 12** Take an image of a scene with straight lines, resample it according to the given equation. Find  $\gamma$  that makes the lines straight in the image too.

**Exercise 13**  $\alpha/\beta = 1.1$  (and  $\gamma = 0$  if it is a perfect rectangle).

**Exercise 14**  $\mathbf{H} = \mathbf{K}[\mathbf{R} \ \mathbf{t}]$  is determined up to 8 parameters by 1 calibration plane. There are 6 degrees of freedom in  $[\mathbf{R} \ \mathbf{t}]$ , 3 rotation angles and 3 translation directions. Consequently  $8-6=2$  equations are obtained for solving  $\mathbf{K}$  from one calibration plane. Since there are 5 unknowns in  $\mathbf{K}$ , at least 3 calibration planes are needed.

**Exercise 15** **Example I:** finding corresponding points in stereo images. The epipolar constraint can be used to check if two points, one from each image, correspond to the same 3D point. In practice this have to be done by checking if either of the two points lie close the epipolar line generated by the other point. If this is not the case, the two points are not corresponding to the same 3D point. If the distance to epipolar lines is within a reasonable distance (given by the measurement noise), the two point may be corresponding but this is not guaranteed. They can be projection of two 3D points that happen to lie in the same epipolar plane. **Example II:** If a set of (at least 8 pairs of) corresponding points have been detected, they can be used to compute the fundamental matrix  $\mathbf{F}$ , using the 8-point algorithm. **Example III:** Rectification of stereo images is based on knowing the fundamental matrix  $\mathbf{F}$ .

**Exercise 16** See the figure in the exam. The point  $p_i = (w_i/2, 0, f)^T$  and  $p_r = s(c_x + w/2, c_y, 1)^T$ . Therefore

$$s \begin{pmatrix} c_x + w/2 \\ c_y \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & 0 & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_i/2 \\ 0 \\ f \end{pmatrix}. \quad (1)$$

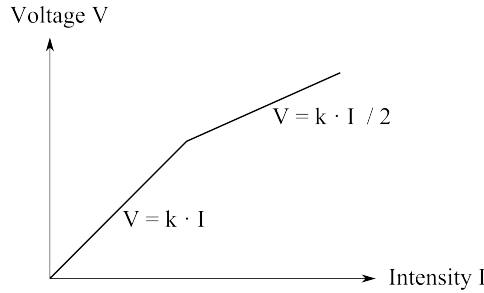
The first row gives  $s(c_x + w/2) = \alpha w_i/2 + c_x f$  and the second row gives  $s c_y = c_y f$  and the third row gives  $s = f$ . Therefore  $f(c_x + w/2) = \alpha w_i/2 + c_x f$ , which gives  $w_i = w f / \alpha$ .

Finally,  $\theta_{\text{FOV}} = 2 \arctan(w_i/(2f)) = 2 \arctan(w/(2\alpha))$ .

Similarly,  $\phi_{\text{FOV}} = 2 \arctan(h/(2\beta))$ .

## PART III: NON-STANDARD IMAGE SENSORS

**Exercise 17**



**Exercise 18** To enter the camera, part of the reflected light must be directed towards the camera. A diffuse reflecting surface reflects light equally in all directions, whereas a specular reflecting surface reflects light in a specular lobe around the direction of perfect reflection. Therefore, there is a risk that the reflected light from a specular reflecting surface might miss the camera.

**Exercise 19** If the reflected laser pulse comes from a border area which is dark on one side and bright on the other side, the detected pulse shape becomes distorted. Then there will be a slight change in detected max position which will show up as a height difference in the range image. The contrast between letter and background is larger in the “K” card than in the “M” card. Therefore, the range deviations will be larger for the “K” card.

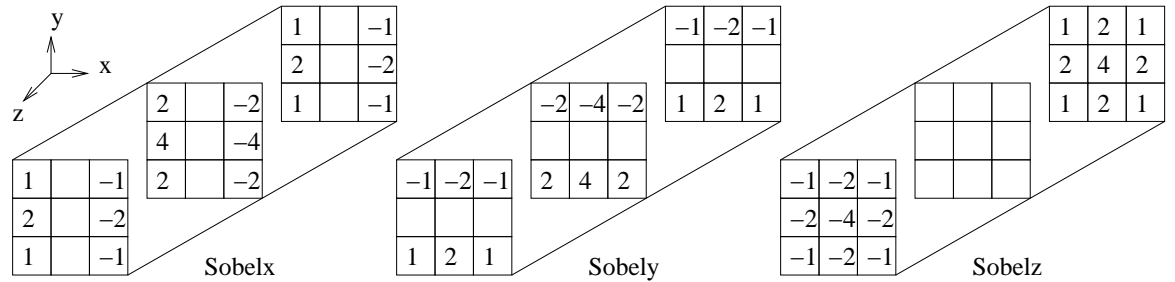
**Exercise 20** Gray coded patterns are binary patterns that are projected on the object. Gray coded patterns are more robust against large errors than common binary coded patterns, but otherwise they work in a similar way. By using  $N$  patterns,  $2^N$  unique positions can be obtained in one of the two directions.  $2^9 = 512$  and therefore 9 patterns are needed.

**Exercise 21** In CT, there is a rule that  $N_\phi = (\pi/2) \cdot N_r$  gives a good image quality and increasing  $N_\phi$  will have almost no effect. Decreasing  $N_\phi$ , however, will gradually destroy the image quality. For very low values of  $N_\phi$ , streaks originating from backprojection will be clearly visible.  $N_\phi = (\pi/4) \cdot N_r$  i.e. reducing  $N_\phi$  to half the number of desired projection angles, will have the effect:

- slightly worse, streaks are beginning to show up

**Exercise 22** A MIP projects only the voxels with *maximum intensity* that fall in the way of parallel rays traced from the viewpoint to the plane of projection.

**Exercise 23**



$$\frac{\partial f(x, y, z)}{\partial x} = f(x, y, z) * \text{sobelx},$$

$$\frac{\partial f(x, y, z)}{\partial y} = f(x, y, z) * \text{sobely},$$

$$\frac{\partial f(x, y, z)}{\partial z} = f(x, y, z) * \text{sobelz}.$$

$$\text{grad}(x, y, z) = \left( \frac{\partial f(x, y, z)}{\partial x}, \frac{\partial f(x, y, z)}{\partial y}, \frac{\partial f(x, y, z)}{\partial z} \right)$$

$$\bar{n}(x, y, z) = -(\text{grad}(x, y, z) / |\text{grad}(x, y, z)|)$$

**Exercise 24**

- a)  $f(x, y) = 3 \cdot \Pi(x/4) \cdot \Pi(y/2) \Rightarrow$   
 $F(u, v) = 3 \cdot 4\text{sinc}(4u) \cdot 2\text{sinc}(2v) = 24\text{sinc}(4u)\text{sinc}(2v)$   
 $p(r, 0) = 2 \cdot 3 \cdot \Pi(r/4) = 6\Pi(r/4) \Rightarrow$   
 $P(R, 0) = 24\text{sinc}(4R)$
- b) Proof of the Projection theorem for  $\theta = 0$ :  
 $F(R \cos 0, R \sin 0) = F(R, 0) = 24\text{sinc}(4R) \cdot \text{sinc}(0) = 24\text{sinc}(4R) = P(R, 0)$   
Q.E.D.