

TSBB21, Lecture 3

3D visualization

p. 1

Visualization of 3D-volumes:

- Look-through projections
- Depth coding
- Surface shading
- Rotation
- Stereo
- MIP (Maximum Intensity Projections)
- Emission/absorption
- Compositing and transfer functions

Introduction to the computer exercise

Lab

Figure references to:

- Engel: Real-Time Volume Graphics.
- Appendix of Computer Exercise in 3D Visualization Laboratory

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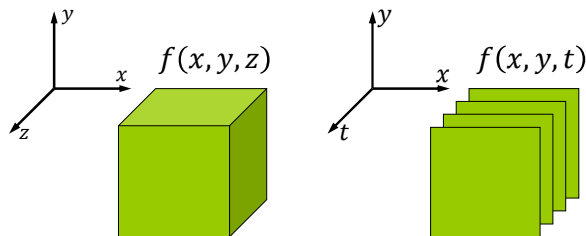
Why?

p. 2

- We will use some of the light and surface models from Lecture 1:
 - Surface reflection
 - Surface models: mirror, diffuse, etc.
 - Absorption
 - Emission
 - Color
- Some instruments, like computed tomography, CT, give 3D volumes. To let the viewer get a 3D impression of these volumes, it is common to visualize them.
- Visualization of 3D volumes is different from computer graphics but has many similarities.

Signals in 3 dimensions

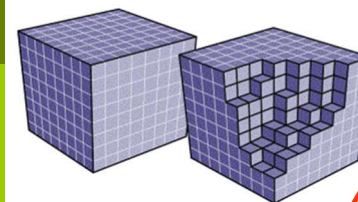
p. 3



2D-image	3D-volume	3D-image sequence	4D
$f(x, y)$ pixel intensity	$f(x, y, z)$ voxel density/ intensity	$f(x, y, t)$ pixel intensity	$f(x, y, z, t)$ voxel density/ intensity

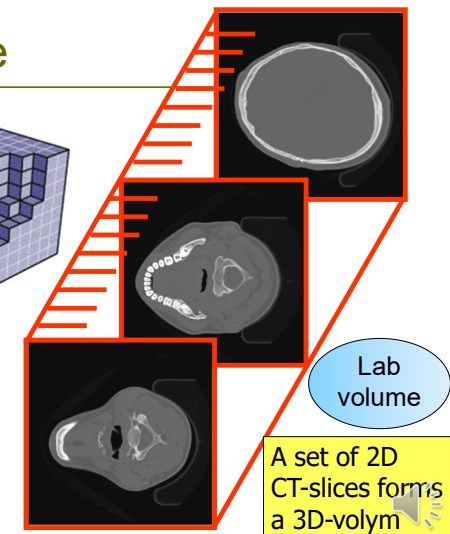
A 3D Volume

p. 4



Volume data set
given on a discrete
uniform grid

Fig. 1.2



A set of 2D
CT-slices forms
a 3D-volym

Instruments that give 3D-volumes

p. 5

- ❑ Computed tomography (CT) (X-rays)
- ❑ Magnetic resonance imaging (MRI) (nuclear spin resonance)
- ❑ Ultrasound (normally 2.5D, only)
- ❑ Confocal microscopy
- ❑ Transmission electron microscopy with tomography
- ❑ Gamma cameras with tomography (gamma-rays)
- ❑ PET (positron electron tomography) (gamma-rays)
- ❑ Mechanical slicing and photography. Different preparations from biology and material technique



The visualization problem

p. 6

We cannot see a 3D-function.
On our retina, a 2D-function is projected.

- ❑ Visualization: A mapping from 3D to 2D
 - 1. look-through and X-ray projections
 - 2. MIP (Maximum Intensity Projections)
 - 3. Depth coding
 - 4. Surface shading with e.g. the Phong model
 - 5. Emission/absorption, Compositing (where 4. can be included, too)
- ❑ Improved visualization with:
 - a. rotation
 - b. stereo



Look-through projections

p. 7

$$p_{\phi}(x, z) = \int f_{\phi}(x, y, z) dy \quad \text{Eq. (1)}$$

$$p_{\phi}(x, z) = \sum_y f_{\phi}(x, y, z) \quad \text{Eq. (2)}$$

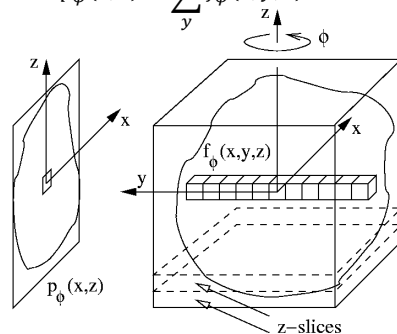


Figure 1

X-ray projections

p. 8

- ❑ To get X-ray projections, compute

$$I_0 - I_{\phi}(x, z) = I_0 - I_0 \exp \left[- \int f_{\phi}(x, y, z) dy \right] \quad \text{Eq. (3)}$$
- ❑ where $f_{\phi}(x, y, z)$ now corresponds to the X-ray attenuation coefficient (absorption) of the matter,
- ❑ I_0 is the incident X-ray intensity and $I_0 - I_{\phi}(x, z)$ is the conventional X-ray image.

See image at p. 39



Other ways to move through a volume

p. 9

More on p. 30-32!

- Ray-casting for a perspective projection

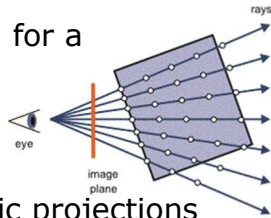


Fig. 1.11

- Orthographic projections

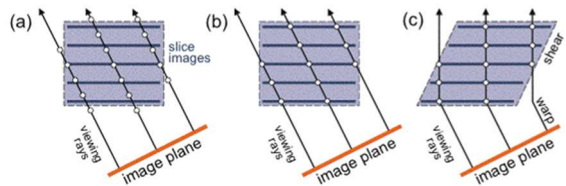


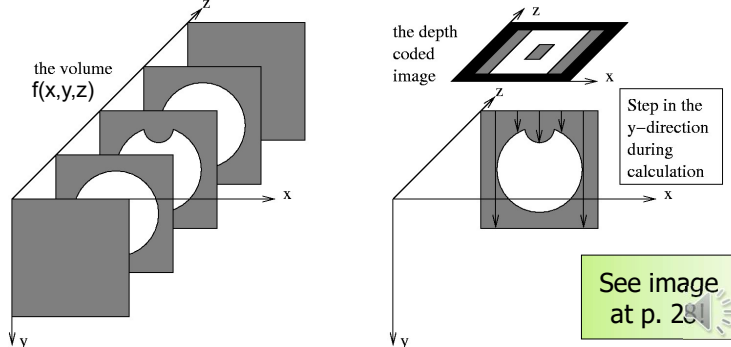
Fig. 1.12

Depth coding

p. 10

Lab!

- Parallel rays are sent through the volume
- Note the y-value where $f(x,y,z) > T$
- Set the intensity to N-y



Radiance I and Radiative energy Q

p. 11

- Radiance is defined as radiative energy Q per projected unit area A , per solid angle Ω and per unit of time t :

$$I = \frac{dQ}{dA_{\perp} d\Omega dt}$$

Eq. (1.1)

Eq. (5.2)

Common types of light sources

p. 12

Sec. 5.2

- Point light sources
 - Emit light at a single point in space equally in all directions. A fall-off function specifies the reduction of intensity with respect to distance from the light source (proportional to r^2).
- Directional light sources
 - Point light sources at infinity. They are solely described by their light direction, and all emitted light rays are parallel to each other. Ex) The sun.

Calculating the normal vector of an iso-surface

p. 13

The 3D gradient

$$\nabla f(\mathbf{x}) = \begin{pmatrix} \frac{\partial f(\mathbf{x})}{\partial x} \\ \frac{\partial f(\mathbf{x})}{\partial y} \\ \frac{\partial f(\mathbf{x})}{\partial z} \end{pmatrix}$$

Eq. (5.4)

The normal vector

$$\mathbf{n}(\mathbf{x}) = \frac{\nabla f(\mathbf{x})}{\|\nabla f(\mathbf{x})\|}, \quad \text{if } \|\nabla f(\mathbf{x})\| \neq 0$$

Eq. (5.5)



Gradient estimation

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Central differences

$$\nabla f(\mathbf{x}) \approx \frac{1}{2} \begin{pmatrix} f(x+1, y, z) - f(x-1, y, z) \\ f(x, y+1, z) - f(x, y-1, z) \\ f(x, y, z+1) - f(x, y, z-1) \end{pmatrix} \quad \text{Eq. (5.15)}$$

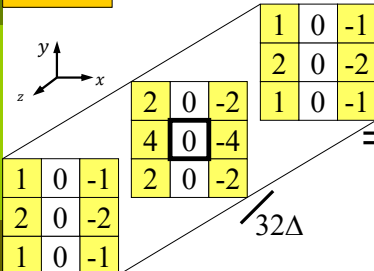


Derivative estimation with 3D Sobel filters

(2D sobel filters in e.g. TSBB08 or TSBB31)

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Sobelx:



$$\frac{\partial f(\mathbf{x})}{\partial x} \approx \text{sobelx} * f(\mathbf{x})$$

Compare Eq. (5.16)

- $[1, 2, 1]/4$ performs low-pass filtering in the z-direction.
- $[1, 0, -1]/2\Delta$ derivatives and performs low-pass filtering in the x-direction.
- $[1, 2, 1]/4$ performs low-pass filtering in the y-direction.



Computational complexity

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Kernel	Dimension	Number of operations			
		without separation		with separation	
		MUL	ADD/SUB	MUL	ADD/SUB
2D: sobelx	3x3	2	5	1	3
3D: sobelx	3x3x3	10	17	2	5

Due to increased bug risk:
Avoid separation in Lab!



Illumination models

- ▣ The Phong model (or the similar, but more efficient Blinn-Phong model)

$$I_{Phong} = I_{ambient} + I_{diffuse} + I_{specular} \quad (Eq. 5.17)$$

- ▣ The Volume model

$$I_{volume} = I_{emission} + I_{Phong} \quad (Eq. 5.27)$$

The emission-absorption model, see later



The ambient term

- ▣ The ambient term compensates for missing indirect illumination and lights up shadows.
- ▣ It has no practical justification.

$$I_{ambient} = k_a M_a I_a \quad (Eq. 5.18)$$

Light (RGB color)

coefficient between 0 and 1

Material (RGB color)



Different types of reflections

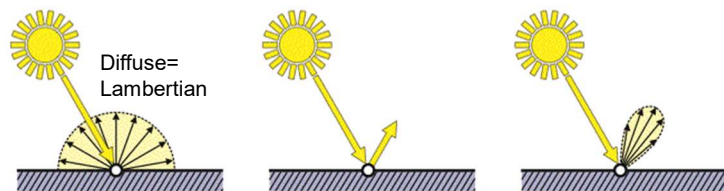


Figure 5.3. Different types of reflections. Left: *Lambertian* surfaces reflect light equally in all directions. Middle: *perfect mirrors* reflect incident light in exactly one direction. Right: shiny surfaces reflect light in a *specular lobe* around the direction of perfect reflection (*specular reflection*).



Geometry for reflection

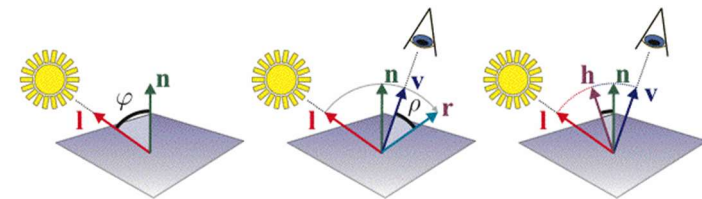


Figure 5.4. Left: the diffuse illumination term depends on the angle of incidence φ between the normal \mathbf{n} and the light direction \mathbf{l} . Middle: in the original Phong model, the specular term is based on the angle ρ between the reflected light vector \mathbf{r} and the viewing direction \mathbf{v} . Right: the specular term depends on the angle between the normal and a vector \mathbf{h} , which is halfway between the viewing and the light direction.



The diffuse term

- The diffuse term corresponds to Lambertian reflection, which means that light is reflected equally in all directions.
- Depends only on the angle of incidence.

$$\begin{aligned} \mathbf{I}_{diffuse} &= k_d \mathbf{M}_d \mathbf{I}_d \cos \varphi, & \text{if } \varphi \leq \pi/2, & \quad (\text{Eq. 5.19}) \\ &= k_d \mathbf{M}_d \mathbf{I}_d \max(\mathbf{l} \cdot \mathbf{n}, 0) & & \quad (\text{Eq. 5.20}) \end{aligned}$$

Light (RGB color) → \mathbf{I}_d

coefficient between 0 and 1 → k_d

Material (RGB color) → \mathbf{M}_d



The specular term

- The specular term models shiny surfaces.
- Depends also on the viewing angle.
- It is a phenomenological model. It produces realistic effects, but certain aspects are not physically plausible.

$$\begin{aligned} \mathbf{I}_{specular} &= k_s \mathbf{M}_s \mathbf{I}_s \cos^n \rho, & \text{if } \rho \leq \pi/2, & \quad (\text{Eq. 5.23}) \\ &= k_s \mathbf{M}_s \mathbf{I}_s (\mathbf{r} \cdot \mathbf{v})^n, & \text{if } \rho \leq \pi/2, & \quad (\text{Eq. 5.24}) \end{aligned}$$

Light (RGB color) → \mathbf{I}_s

Controls shininess → n

coefficient between 0 and 1 → k_s

Material (RGB color) → \mathbf{M}_s



Cook&Torrance – A physically based specular model

$$\mathbf{I}_{specular} = k_s \mathbf{M}_s \mathbf{I}_s \frac{F \cdot D \cdot G}{(\mathbf{n} \cdot \mathbf{v})}, \quad (5.30)$$

consists of a Fresnel term F , a statistical distribution D that describes the orientation of the microfacets, and a geometric self-shadowing term G :

$$F \approx (1 + (\mathbf{v} \cdot \mathbf{h}))^4; \quad (5.31)$$

$$D \approx C \cdot \exp\left(\frac{(\mathbf{h} \cdot \mathbf{n})^2 - 1}{m}\right); \quad (5.32)$$

$$G = \min\left(1, \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{v})}{(\mathbf{h} \cdot \mathbf{v})}, \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{l})}{(\mathbf{h} \cdot \mathbf{v})}\right). \quad (5.33)$$



Blinn-Phong and Cook-Torrance give similar results

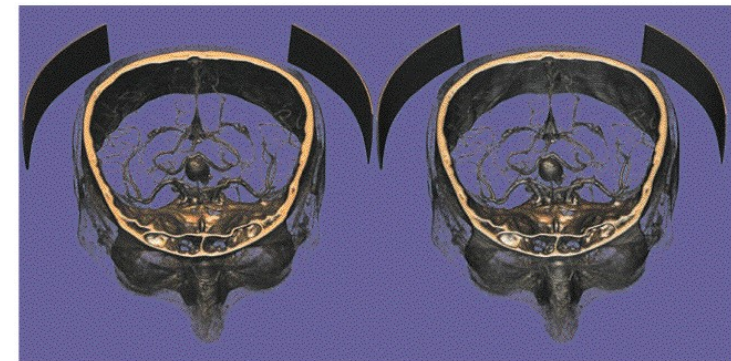
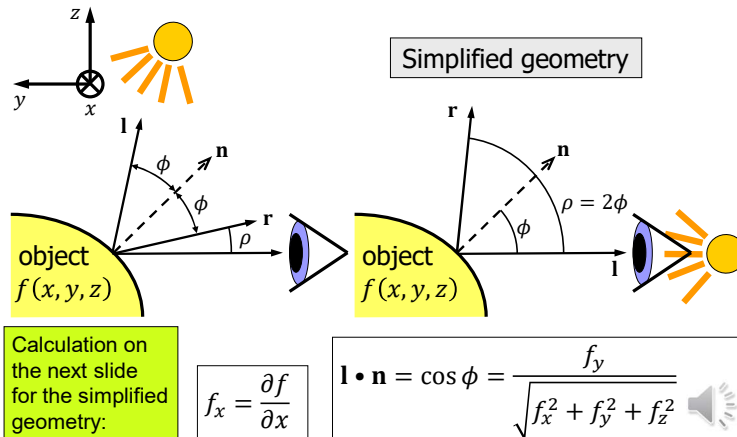


Figure 5.5. Examples of gradient-based volume illumination using the local illumination models by Blinn-Phong (left) and Cook-Torrance (right)



Surface shading with the Phong model

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Calculation of $\cos \phi$

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See previous slide:

- $\bar{\mathbf{l}} = (0, -1, 0)$
- surface gradient $= \nabla f = (f_x, f_y, f_z)$
- $\bar{\mathbf{n}} = \frac{-\nabla f}{|\nabla f|}$ (object density > air)

$$\cos \phi = \bar{\mathbf{l}} \cdot \bar{\mathbf{n}} = (0, -1, 0) \cdot \frac{(-f_x, -f_y, -f_z)}{|\nabla f|}$$

$$= \frac{f_y}{\sqrt{f_x^2 + f_y^2 + f_z^2}}$$

Lab!

More about the Phong model

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- A diffuse (Lambertian) surface reflects the light equally in all directions.
 - Example: Household roll
 - Note that the diffuse reflection is not dependent on the viewing direction ρ .
 - A surface that is slanted in relation to the light source look darker than a surface that is orthogonal to the light source. This is because the slanted surface is less illuminated.
- The specular term in the Phong model is experimentally evaluated to mimic a shiny surface.
 - Example: Steel thermos
 - A perfectly shiny surface has an infinitely large n .
 - In practice, a lower n , e.g. $n=5$, is suitable for a shiny surface.

Example images

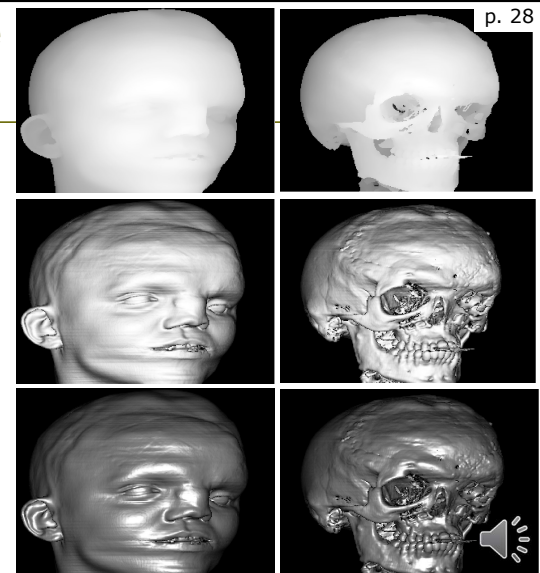
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Depth coding:

Lab!

Diffuse reflection:

Diffuse + specular reflection:



The Phong model and color

$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}} \quad (\text{Eq. 5.17})$$

$$\mathbf{I}_{\text{ambient}} = k_a \mathbf{M}_a \mathbf{I}_a \quad (\text{Eq. 5.18})$$

$$\mathbf{I}_{\text{diffuse}} = k_d \mathbf{M}_d \mathbf{I}_d \cos \phi, \quad \text{if } \phi \leq \pi/2, \quad (\text{Eq. 5.19})$$

$$= k_d \mathbf{M}_d \mathbf{I}_d \max(\mathbf{l} \cdot \mathbf{n}, 0) \quad (\text{Eq. 5.20})$$

$$\mathbf{I}_{\text{specular}} = k_s \mathbf{M}_s \mathbf{I}_s \cos^n \rho, \quad \text{if } \rho \leq \pi/2, \quad (\text{Eq. 5.23})$$

$$= k_s \mathbf{M}_s \mathbf{I}_s (\mathbf{r} \cdot \mathbf{v})^n, \quad \text{if } \rho \leq \pi/2, \quad (\text{Eq. 5.24})$$

- Both the light \mathbf{I} and material \mathbf{M} contain RGB-information. Note that they are multiplied.

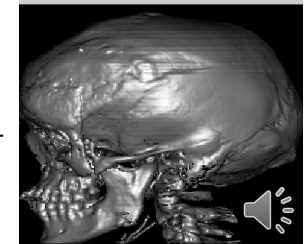


Rotation

- To obtain the impression of rotation, do like this:

- Compute a sequence of projection images with different angles and angular difference $> \approx 1^\circ$.
- Show the images on the screen, one at a time in a suitable speed.
- Our brain gets the impression of a rotating object volume.

The film shows 46 views with 4° angular difference

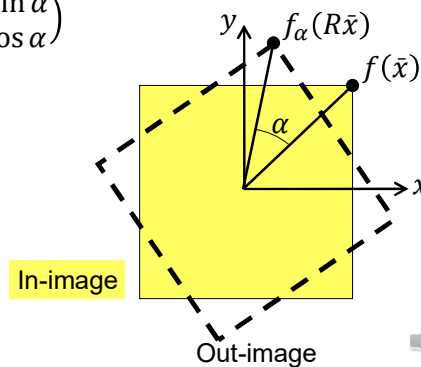


Rotation in 2D, repetition

(e.g. from TSBB08, TSBB31)

$$R = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

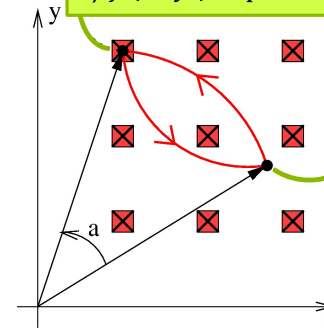
$$\bar{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$



Rotation in 2D, repetition Inverse mapping

- (x, y) is given
- $f(x', y')$ is placed in (x, y)

Observe!
The in-image \blacksquare
and the
out-image \times
are overlaid.



- (x', y') is calculated
 $f(x', y')$ is interpolated

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = R^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

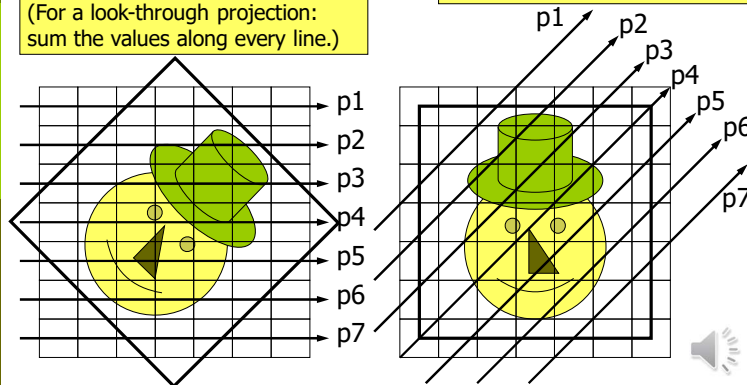


Calculation of a rotated projection

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Alternative 1: Rotate every slice. Step forward along straight lines. (For a look-through projection: sum the values along every line.)

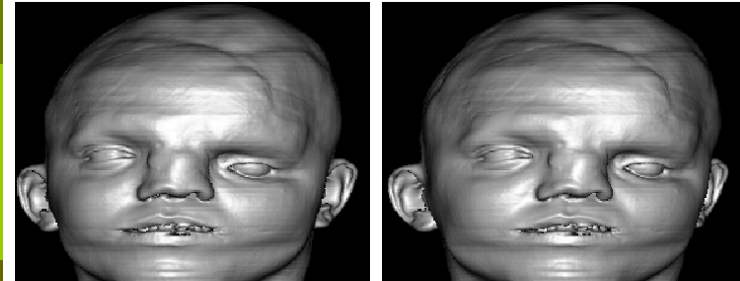
Alternative 2: Ray casting, see also slide p. 9. Step forward along slanted rays in the slice with \approx pixel distance. If necessary, interpolate the pixel values.



A stereo pair of surface shaded images

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Lab!

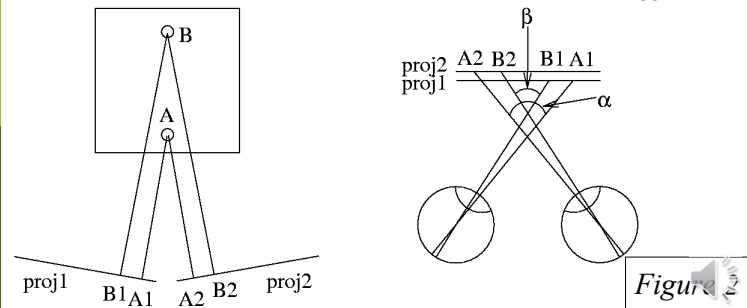


- Look at the figure with a crossed gaze. It may be helpful to hold your finger between your eyes and the stereo pair.
- After a while, you can see three blurred images.
- Take away the finger, concentrate on the middle image and try to make it sharp and steady.

Computation of stereo images

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- Compute 2 projections with angular difference $\approx 8^\circ$.
- Show proj1 to the left eye and proj2 to the right eye.
- The brain gets the impression of looking at a 3D scene with two eyes. It matches $A1-A2$ and $B1-B2$. $A1-A2 = \alpha$, $B1-B2 = \beta$. $\alpha > \beta \Rightarrow$ the brain believes that A is nearest.



Stereo with red-green glasses

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Lab!

Extra exercise



Examples of more modern glasses for 3D television

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Functional principle
of active shutter 3D
systems.

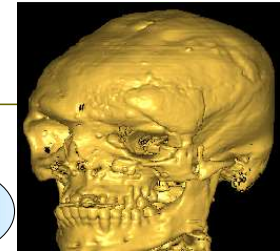


Functional principle
of polarized 3D
systems.

Exercise: Produce a golden skull

- This material data can be used in the Phong model to simulate different materials:

Lab



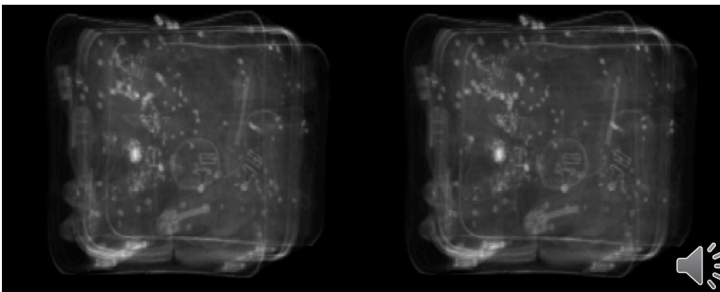
Name	Ambient			Diffuse			Specular			Shininess
emerald	0.0215	0.1745	0.0215	0.07568	0.61424	0.07568	0.633	0.727811	0.633	32*0.6
jade	0.135	0.2225	0.1575	0.54	0.89	0.63	0.316228	0.316228	0.316228	32*0.1
obsidian	0.05375	0.05	0.06625	0.18275	0.17	0.22525	0.332741	0.328634	0.346435	32*0.3
pearl	0.25	0.20725	0.20725	1	0.829	0.829	0.296648	0.296648	0.296648	32*0.088
ruby	0.1745	0.01175	0.01175	0.61424	0.04136	0.04136	0.727811	0.626959	0.626959	32*0.6
turquoise	0.1	0.18725	0.1745	0.396	0.74151	0.69102	0.297254	0.30829	0.306678	32*0.1
brass	0.329412	0.223529	0.027451	0.780392	0.568627	0.113725	0.992157	0.941176	0.807843	32*0.21794872
bronze	0.2125	0.1275	0.054	0.714	0.4284	0.18144	0.393548	0.271906	0.166721	32*0.2
chrome	0.25	0.25	0.25	0.4	0.4	0.4	0.774597	0.774597	0.774597	32*0.5
copper	0.19125	0.0735	0.0225	0.7038	0.27048	0.0828	0.256777	0.137622	0.086014	32*0.1
gold	0.24725	0.1995	0.0745	0.75164	0.60648	0.22648	0.628281	0.555802	0.366065	32*0.4
silver	0.19225	0.19225	0.19225	0.50754	0.50754	0.50754	0.508273	0.508273	0.508273	32*0.4

A stereo pair of X-ray projections of a bag

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Figure 3

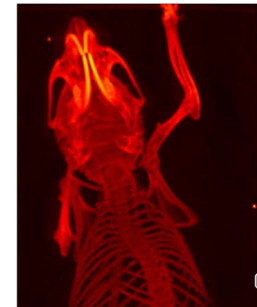
- Look at the figure with a crossed gaze. It may be helpful to hold your finger between your eyes and the stereo pair.
- After a while, you can see three blurred images.
- Take away the finger, concentrate on the middle image and try to make it sharp and steady.



MIP (Maximum intensity projection)

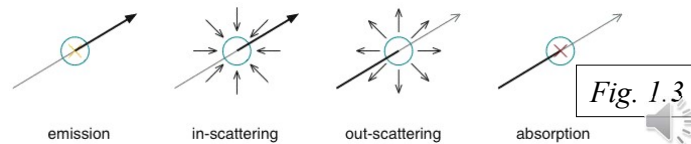
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- Projects only the voxels with maximum intensity that fall in the way of parallel rays traced from the viewpoint to the plane of projection.
- To help the viewer's perception, rotation can be added to the MIP of the mouse here:
- Note that a MIP projection at angle 0° and angle 180° look the same, which can hinder the perception.
- An easy improvement to MIP is "Local maximum intensity projection". In this technique we don't take the global maximum value, but the first maximum value that is above a certain threshold.



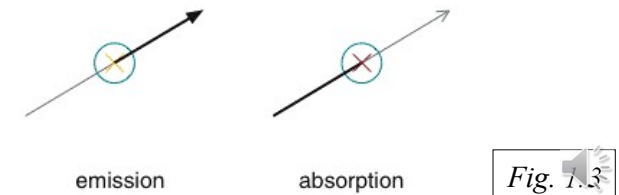
Physical model of light transport

- Emission (see next slide)
- Absorption (see next slide)
- Scattering
 - Light can be scattered by participating media, essentially changing the direction of light propagation. If the wavelength is not changed by scattering, the process is called elastic scattering. Conversely, inelastic scattering affects the wavelength.



Emission and absorption (Scattering is now not considered.)

- Emission
 - The gaseous material emits light, increasing the radiative energy. In reality, for example, hot gas emits light by converting heat into radiative energy.
- Absorption
 - Material can absorb light by converting radiative energy into heat. In this way, light energy is reduced.



Ex) with emission/absorption, yellow bone and pink skin

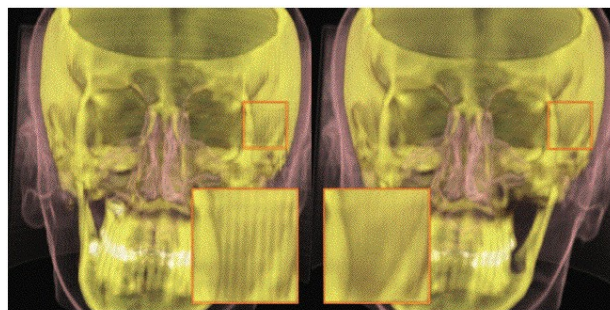


Figure 1.10. Comparison between trilinear filtering (left) and cubic B-spline filtering (right).

Ex) An advanced Visualization of functional MRI (fMRI, LiU)

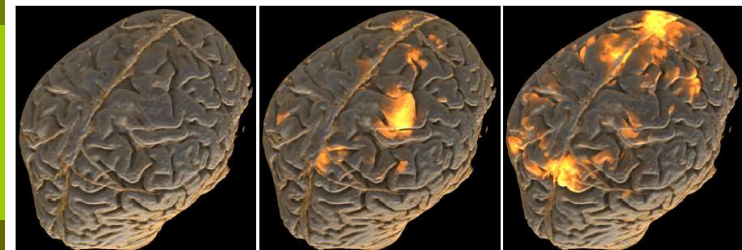


Figure 4: *Left:* The anatomy is rendered using the local ambient occlusion shading model, the A^L reflective light, which enhances the perception of depth. A diffuse shading component is also applied. *Middle:* The fMRI signal rendered using the LAO emission, i.e. including A^E . The image represents the brain activation during repeated motion of the left foot. *Right:* The activation during mathematical problem solving. The instructions, as well as the visualization of the brain activity as shown in these images, are shown to the subject in a head mounted display, and thus there is significant activation of the visual cortex.

Ex) An advanced Visualization of functional MRI (IMT, LiU). Details.

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- A high resolution volume of the brain is measured with an MRI-scanner.
- This volume is visualized by diffuse surface shading.
- A low resolution 4D time sequence of the brain is registered and correlated with some activity, e.g. motion of the left foot. Activated brain area = increased blood flow.
- The low resolution detected activity is visualized as a light emitting area in the brain.



Related to absorption: Beer's Law (or Beer-Lambert Law)

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- Intensity: I
- Linear attenuation coefficient: μ [1/m]

Diagram illustrating Beer's Law:

Top diagram: A small yellow box labeled μ with a width Δy . An arrow labeled I enters from the left, and an arrow labeled $I + \Delta I$ exits to the right.

Bottom diagram: A larger yellow box labeled μ with a width L . An arrow labeled I_{in} enters from the left at $y=0$, and an arrow labeled I_{out} exits to the right at $y=L$.

Equations:

$$\Delta I = -I \cdot \mu \cdot \Delta y \quad \Rightarrow \quad \text{differentiate} \quad dI = -I \cdot \mu \cdot dy$$

$$\Rightarrow \text{integrate} \quad \int_{I_{in}}^{I_{out}} \frac{dI}{I} = \int_0^L -\mu \, dy \Rightarrow [\ln I]_{I_{in}}^{I_{out}} = [-\mu \cdot y]_0^L \Rightarrow$$

$$\ln I_{out} - \ln I_{in} = -\mu L \Rightarrow I_{out} = I_{in} e^{-\mu L}$$

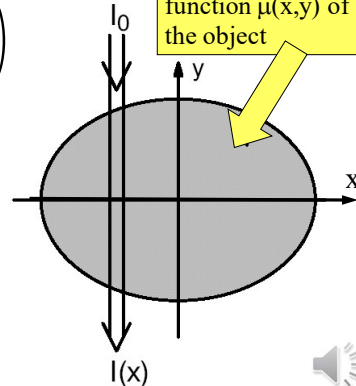


The attenuation/absorption of light or X-rays in an object

p. 47

$$I(x) = I_0 \exp \left(- \int_{-\infty}^{\infty} \mu(x, y) \, dy \right)$$

The attenuation function $\mu(x, y)$ of the object

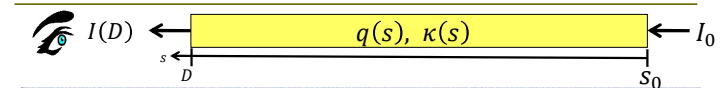


Compare with the "Volume-Rendering Integral" Eq.(1.7) on the next slide.



Volume rendering integral

p. 48



Volume-Rendering Integral

The emission-absorption optical model leads to the volume-rendering integral:

$$I(D) = I_0 e^{-\int_{s_0}^D \kappa(t) \, dt} + \int_{s_0}^D q(s) e^{-\int_s^D \kappa(t) \, dt} \, ds, \quad (1.7)$$

with optical properties κ (absorption coefficient) and q (source term describing emission) and integration from entry point into the volume, $s = s_0$, to the exit point toward the camera, $s = D$.

Radiance leaving the volume

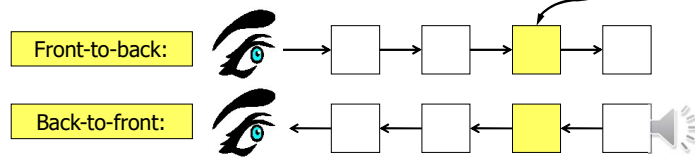
Light entering from the background



Using Compositing to compute the volume rendering integral

p. 49

- Each pixel gives an intensity C_i , possibly in color, and an opacity value α_i .
- With this technique, voxels with high α_i -value can dominate, shadow, behind-lying pixels and light through forward-lying pixels.
- Composition can be performed Front-to-back or Back-to-front.
- I cannot follow the description in the book. Does it contain errors? On the next pages, I give you an alternative description.



Back-to-front compositing

p. 50

- Assume that each sample on a view ray has color and opacity: $(C_0, \alpha_0), \dots, (C_b, \alpha_b)$, $C_i \in [0,1]^3$, $\alpha_i \in [0,1]$ where the 0th sample is next to the camera and the bth one is a (fully opaque) background sample: $C_b = (r, g, b)_{background}$ $\alpha_b = 1$
- Compositing can be defined recursively: Let C_f^b denote the **composite color** of samples $f, f+1, \dots, b$. Recursion formula for **back-to-front** compositing:

$$C_b^b = \alpha_b \cdot C_b$$

$$C_i^b = \alpha_i \cdot C_i + (1 - \alpha_i) \cdot C_{i+1}^b$$

Diagram illustrating the back-to-front compositing process. A camera on the left looks at a sequence of pixels (white, white, white, yellow, white, white) from left to right. A yellow box labeled C_i, α_i is shown above the yellow pixel. A red box labeled C_b, α_b is shown above the last white pixel. Red arrows indicate the flow of information: 'own color' from the yellow pixel and 'color from behind' from the composite of pixels to its right. A red box labeled 'transparency' is shown above the yellow pixel.

Back-to-front compositing

p. 51

- The first few generations, written with **transparency** $T_i = 1 - \alpha_i$

$$C_b^b = \alpha_b C_b$$

$$C_{b-1}^b = \alpha_{b-1} C_{b-1} + \alpha_b C_b T_{b-1}$$

$$C_{b-2}^b = \alpha_{b-2} C_{b-2} + \alpha_{b-1} C_{b-1} T_{b-2} + \alpha_b C_b T_{b-1} T_{b-2}$$

$$C_{b-3}^b = \alpha_{b-3} C_{b-3} + \alpha_{b-2} C_{b-2} T_{b-3} + \alpha_{b-1} C_{b-1} T_{b-2} T_{b-3} + \alpha_b C_b T_{b-1} T_{b-2} T_{b-3}$$

reveal the **closed formula** for compositing:

$$C_f^b = \sum_{i=f}^b \left(\alpha_i C_i \prod_{j=f}^{i-1} T_j \right)$$

Front-to-back compositing

p. 52

- Front-to-back** compositing can be derived from the closed formula: Let T_f^b denote the **composite transparency** of samples $f, f+1, \dots, b$

$$T_f^b = \prod_{j=f}^b T_j$$

- Then the **simultaneous recursion** for front-to-back compositing is:

$$C_f^f = \alpha_f C_f$$

$$T_f^f = 1 - \alpha_f$$

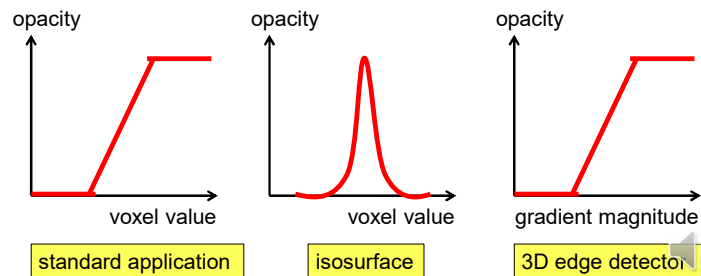
$$C_f^{b+1} = C_f^b + \alpha_{b+1} C_{b+1} T_f^b$$

$$T_f^{b+1} = (1 - \alpha_{b+1}) T_f^b$$

- Advantage of front-to-back compositing: **early ray termination** when composite transparency falls below a threshold

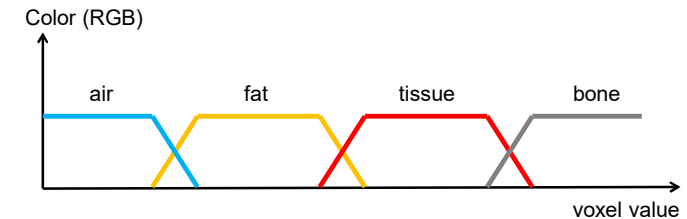
Transfer functions, opacity

- By choosing different opacity transfer functions different types of applications can be achieved.
- Examples:



Transfer functions, color

- The color transfer function allows to make a simple **classification**.
- Example appropriate for a CT-volume:



- Note: Better (but more expensive) classification can be obtained by segmentation.

Volume Ray Casting with compositing and shading

1. Ray casting
 2. Sampling (and interpolation)
 3. Shading
 4. Compositing
- http://en.wikipedia.org/wiki/Volume_ray_casting

