

# Image Sensors

## Lecture H

p. 1

- Visualization of 3D-volumes:
  - Look-through projections
  - Depth coding
  - Surface shading
  - Rotation
  - MIP (Maximum Intensity Projections)
  - Stereo
  - Emission/absorption
  - Compositing and transfer functions
- Introduction to the computer exercise
- Literature
  - Chapter 1 and 5 from the E-book: "Real-Time Volume Graphics" by Klaus Engel, m. fl.
  - The computer exercise document about visualization by Maria Magnusson.

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## Why?

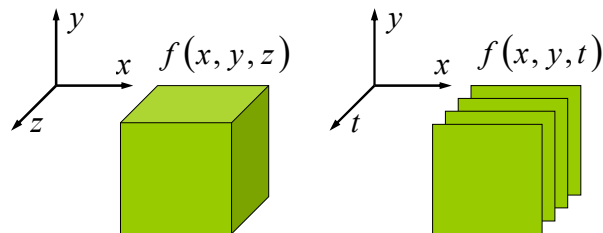
p. 2

- We get use of light and surface models that we have learned previously in the course.
- Some instruments, like computed tomography, CT, which gives 3D-volumes. To let the viewer get a 3D impression of these volumes, it is common to visualize them.
- Visualization of 3D-volumes is different from computer graphics, but has many similarities.



## Signals in 3 dimensions

p. 3

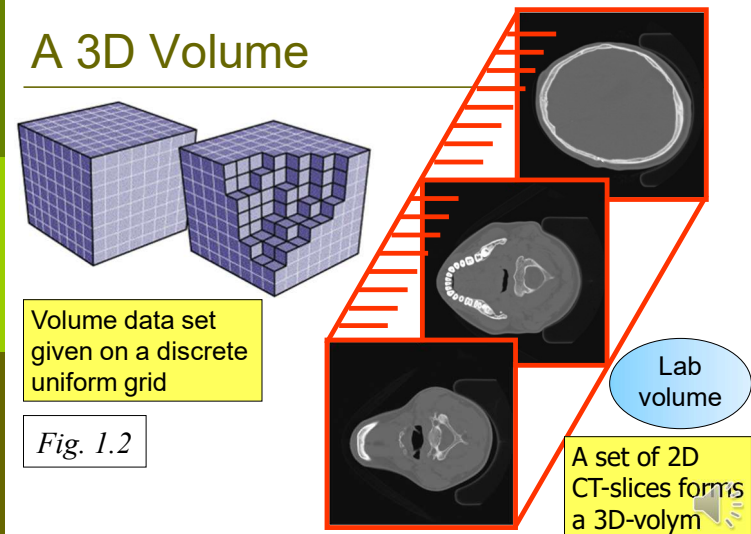


2D-image	3D-volume	3D-image sequence	4D
$f(x, y)$ pixel intensity	$f(x, y, z)$ voxel density/ intensity	$f(x, y, t)$ pixel intensity	$f(x, y, z, t)$ voxel density/ intensity



## A 3D Volume

p. 4



## Instruments that give 3D-volumes

p. 5

- ❑ Computed tomography (CT) (X-rays)
- ❑ Magnetic resonance imaging (MRI) (nuclear spin resonance)
- ❑ Ultrasound (normally 2.5D, only)
- ❑ Confocal microscopy
- ❑ Transmission electron microscopy with tomography
- ❑ Gamma cameras with tomography (gamma-rays)
- ❑ PET (positron electron tomography) (gamma-rays)
- ❑ Mechanical slicing and photography. Different preparations from biology and material technique



## The visualization problem

p. 6

We cannot see a 3D-function.  
On our retina, a 2D-function is projected.

- ❑ Visualization: A mapping from 3D to 2D
  - 1. look-through and X-ray projections
  - 2. MIP (Maximum Intensity Projections)
  - 3. Depth coding
  - 4. Surface shading with e.g. the Phong model
  - 5. Emission/absorption, Compositing (where 4. can be included, too)
- ❑ Improved visualization with:
  - a. rotation
  - b. stereo



## Look-through projections

p. 7

$$p_{\phi}(x, z) = \int f_{\phi}(x, y, z) dy \quad \text{Eq. (1)}$$

$$p_{\phi}(x, z) = \sum_y f_{\phi}(x, y, z) \quad \text{Eq. (2)}$$

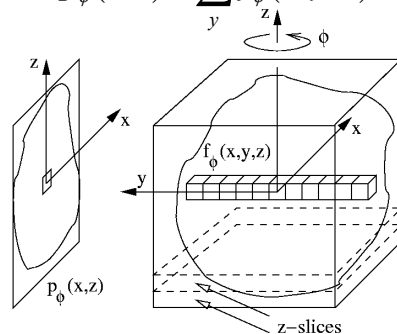


Figure 1

## X-ray projections

p. 8

- ❑ To get X-ray projections, compute
 
$$I_0 - I_{\phi}(x, z) = I_0 - I_0 \exp\left[-\int f_{\phi}(x, y, z) dy\right] \quad \text{Eq. (3)}$$
- ❑ where  $f_{\phi}(x, y, z)$  now corresponds to the X-ray attenuation coefficient of the matter,
- ❑  $I_0$  is the incident X-ray intensity and  $I_0 - I_{\phi}(x, z)$  is the conventional X-ray image.

See image at p. 40



## Other ways to move through a volume

p. 9

More on p. 30-32!

### Ray-casting

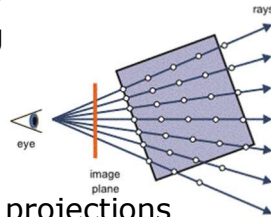


Fig. 1.11

### Orthogonal projections

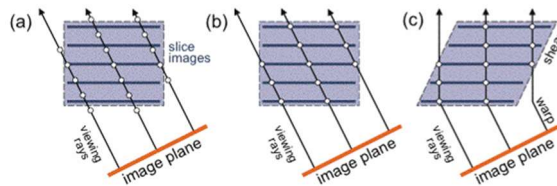


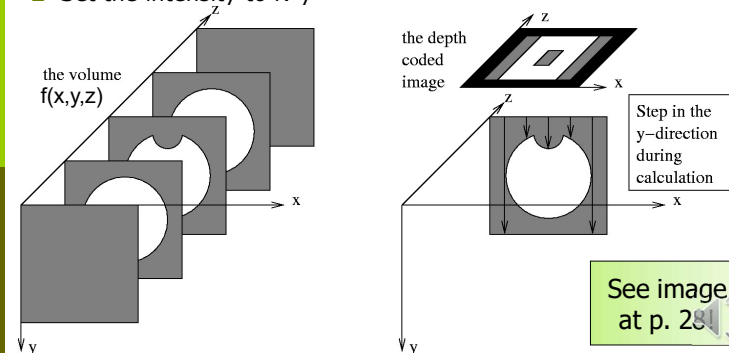
Fig. 1.12

## Depth coding

p. 10

Lab!

- Parallel rays are sent through the volume
- Note the y-value where  $f(x,y,z) > T$
- Set the intensity to N-y



## Radiance I and Radiative energy Q

p. 11

- Radiance is defined as radiative energy  $Q$  per projected unit area  $A$ , per solid angle  $\Omega$  and per unit of time  $t$ :

$$I = \frac{dQ}{dA_{\perp} d\Omega dt}$$

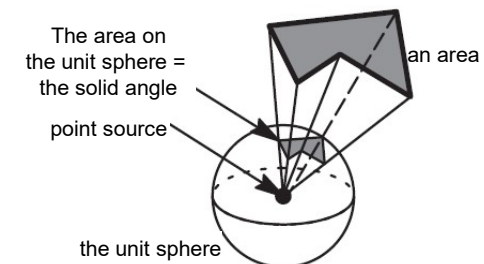
Eq. (1.1)

Eq. (5.2)

## Solid angle

p. 12

- The radiative energy decreases proportional to  $r^2$



$$\text{Maximal solid angle} = \frac{A}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

## Common types of light sources

p. 13

Sec. 5.2

### □ Point light sources

- Emit light at a single point in space equally in all directions. A fall-off function specifies the reduction of intensity with respect to distance from the light source (proportional to  $r^2$ ).

### □ Directional light sources

- Point light sources at infinity. They are solely described by their light direction, and all emitted light rays are parallel to each other. Ex) The sun.



## Calculating the normal vector of an iso-surface

p. 14

The 3D gradient

$$\nabla f(\mathbf{x}) = \begin{pmatrix} \frac{\partial f(\mathbf{x})}{\partial x} \\ \frac{\partial f(\mathbf{x})}{\partial y} \\ \frac{\partial f(\mathbf{x})}{\partial z} \end{pmatrix}$$

Eq. (5.4)

The normal vector

$$\mathbf{n}(\mathbf{x}) = \frac{\nabla f(\mathbf{x})}{\|\nabla f(\mathbf{x})\|}, \quad \text{if } \|\nabla f(\mathbf{x})\| \neq 0$$

Eq. (5.5)



## Gradient estimation

p. 15

Central differences

$$\nabla f(\mathbf{x}) \approx \frac{1}{2} \begin{pmatrix} f(x+1, y, z) - f(x-1, y, z) \\ f(x, y+1, z) - f(x, y-1, z) \\ f(x, y, z+1) - f(x, y, z-1) \end{pmatrix} \quad \text{Eq. (5.15)}$$

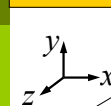


## Derivative estimation with 3D Sobel filters

p. 16

(2D sobel filters in e.g. TSBB08 or TSBB31)

Sobelx:



$$\frac{\partial f(\mathbf{x})}{\partial x} \approx \text{sobelx} * f(\mathbf{x})$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 4 & 0 & -4 \\ 1 & 0 & -1 \end{bmatrix} \cdot \frac{1}{4} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \cdot \frac{1}{2\Delta}$$

Compare Eq. (5.16)

- $[1, 2, 1]/4$  performs low-pass filtering in the z-direction.
- $[1, 0, -1]/2\Delta$  derives and performs low-pass filtering in the x-direction.
- $[1, 2, 1]/4$  performs low-pass filtering in the y-direction.



## Computational complexity

Kernel	Dimension	Number of operations			
		without separation		with separation	
2D: sobelx	3x3	MUL	ADD/SUB	MUL	ADD/SUB
3D: sobelx	3x3x3	2	5	1	3
		10	17	2	5

Due to increased bug risk:  
Avoid separation in Lab!



## Illumination models

- The Phong model (or the similar, but more efficient Blinn-Phong model)

$$\mathbf{I}_{Phong} = \mathbf{I}_{ambient} + \mathbf{I}_{diffuse} + \mathbf{I}_{specular} \quad (Eq.5.17)$$

- The Volume model

$$\mathbf{I}_{volume} = \mathbf{I}_{emission} + \mathbf{I}_{Phong} \quad (Eq.5.27)$$

The emission-absorption  
model, see later



## The ambient term

- The ambient term compensates for missing indirect illumination and lights up shadows.
- It has no practical justification.

$$\mathbf{I}_{ambient} = k_a \mathbf{M}_a \mathbf{I}_a \quad (Eq.5.18)$$

coefficient between 0 and 1

Material (RGB color)

Light (RGB color)



## Different types of reflections

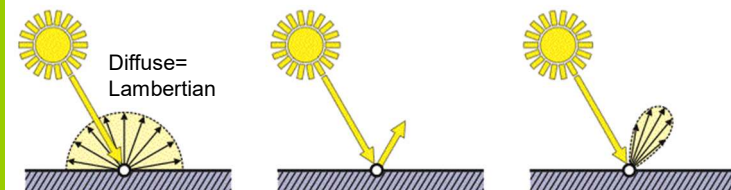


Figure 5.3. Different types of reflections. Left: *Lambertian* surfaces reflect light equally in all directions. Middle: *perfect mirrors* reflect incident light in exactly one direction. Right: shiny surfaces reflect light in a *specular lobe* around the direction of perfect reflection (*specular reflection*).



## Geometry for reflection

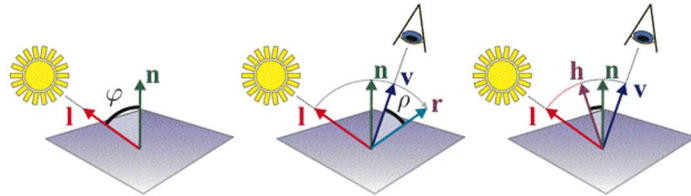


Figure 5.4. Left: the diffuse illumination term depends on the angle of incidence  $\varphi$  between the normal  $\mathbf{n}$  and the light direction  $\mathbf{l}$ . Middle: in the original Phong model, the specular term is based on the angle  $\rho$  between the reflected light vector  $\mathbf{r}$  and the viewing direction  $\mathbf{v}$ . Right: the specular term depends on the angle between the normal and a vector  $\mathbf{h}$ , which is halfway between the viewing and the light direction.



## The diffuse term

- The diffuse term corresponds to Lambertian reflection, which means that light is reflected equally in all directions.
- Depends only on the angle of incidence.

$$\mathbf{I}_{diffuse} = k_d \mathbf{M}_d \mathbf{I}_d \cos \varphi, \quad \text{if } \varphi \leq \pi/2, \quad (\text{Eq. 5.19})$$

$$= k_d \mathbf{M}_d \mathbf{I}_d \max(\mathbf{l} \cdot \mathbf{n}, 0) \quad (\text{Eq. 5.20})$$

Annotations:   
 - "Light (RGB color)" points to  $\mathbf{I}_d$    
 - "coefficient between 0 and 1" points to  $k_d$    
 - "Material (RGB color)" points to  $\mathbf{M}_d$



## The specular term

- The specular term models shiny surfaces.
- Depends also on the viewing angle.
- It is a phenomenological model. It produce realistic effects, but certain aspects are not physically plausible.

$$\mathbf{I}_{specular} = k_s \mathbf{M}_s \mathbf{I}_s \cos^n \rho, \quad \text{if } \rho \leq \pi/2, \quad (\text{Eq. 5.23})$$

$$= k_s \mathbf{M}_s \mathbf{I}_s (\mathbf{r} \cdot \mathbf{v})^n, \quad \text{if } \rho \leq \pi/2, \quad (\text{Eq. 5.24})$$

Annotations:   
 - "Light (RGB color)" points to  $\mathbf{I}_s$    
 - "Controls shininess" points to  $n$    
 - "coefficient between 0 and 1" points to  $k_s$    
 - "Material (RGB color)" points to  $\mathbf{M}_s$



## Cock&Torrance – A physically based specular model

$$\mathbf{I}_{specular} = k_s \mathbf{M}_s \mathbf{I}_s \frac{F \cdot D \cdot G}{(\mathbf{n} \cdot \mathbf{v})}, \quad (5.30)$$

consists of a Fresnel term  $F$ , a statistical distribution  $D$  that describes the orientation of the microfacets, and a geometric self-shadowing term  $G$ :

$$F \approx (1 + (\mathbf{v} \cdot \mathbf{h})^2)^4; \quad (5.31)$$

$$D \approx C \cdot \exp\left(\frac{(\mathbf{h} \cdot \mathbf{n})^2 - 1}{m}\right); \quad (5.32)$$

$$G = \min\left(1, \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{v})}{(\mathbf{h} \cdot \mathbf{v})}, \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{l})}{(\mathbf{h} \cdot \mathbf{v})}\right). \quad (5.33)$$



## Blinn-Phong and Cook-Torrance give similar results p. 25

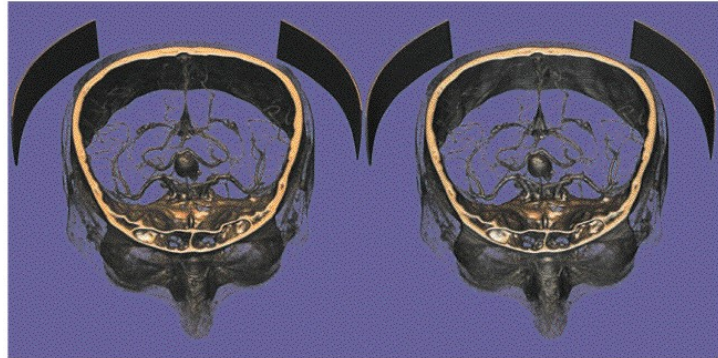
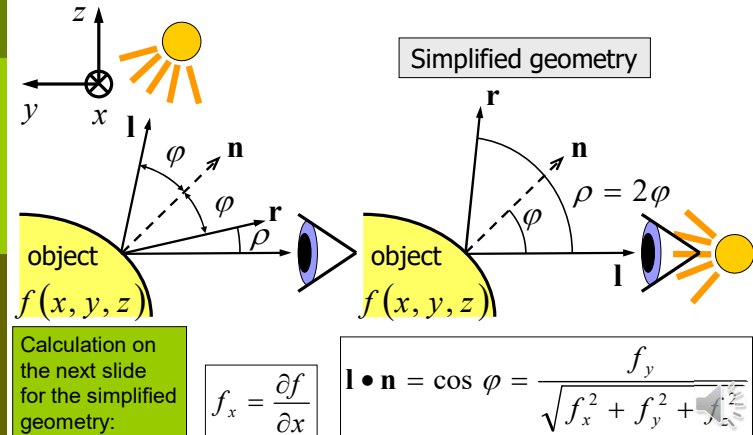


Figure 5.5. Examples of gradient-based volume illumination using the local illumination models by Blinn-Phong (left) and Cook-Torrance (right)

## Surface shading with the Phong model p. 26



## Calculation of $\cos \varphi$ p. 27

■ See previous slide:

- $\bar{\mathbf{l}} = (0, -1, 0)$
- surface gradient  $= \nabla f = (f_x, f_y, f_z)$
- $\bar{\mathbf{n}} = \frac{-\nabla f}{|\nabla f|}$  (object density > air)

$$\cos \varphi = \bar{\mathbf{l}} \cdot \bar{\mathbf{n}} = (0, -1, 0) \cdot \frac{(-f_x, -f_y, -f_z)}{|\nabla f|}$$

$$= \frac{f_y}{\sqrt{f_x^2 + f_y^2 + f_z^2}}$$

Lab!

## More about the Phong model p. 28

- A diffuse (Lambertian) surface reflects the light equally in all directions.
  - Example: Household roll
  - Note that the diffuse reflection is not dependent on the viewing direction  $\rho$ .
  - A surface that is slanted in relation to the light source look darker than a surface that is orthogonal to the light source. This is because the slanted surface is less illuminated.
- The specular term in the Phong model is experimentally evaluated to mimic a shiny surface.
  - Example: Steel thermos
  - A perfectly shiny surface has an infinitely large  $n$ .
  - In practice, a lower  $n$ , e.g.  $n=5$ , is suitable for a shiny surface.



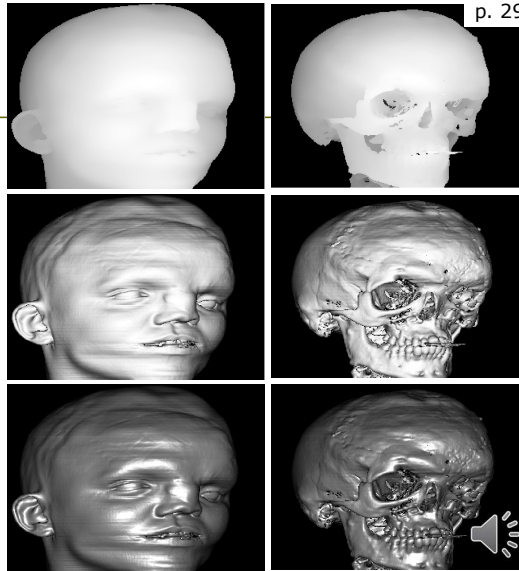
## Example images

Depth coding:

Lab!

Diffuse reflection:

Diffuse + specular reflection:



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## The Phong model and color

$$\mathbf{I}_{Phong} = \mathbf{I}_{ambient} + \mathbf{I}_{diffuse} + \mathbf{I}_{specular} \quad (Eq.5.17)$$

$$\mathbf{I}_{ambient} = k_a \mathbf{M}_a \mathbf{I}_a \quad (Eq.5.18)$$

$$\mathbf{I}_{diffuse} = k_d \mathbf{M}_d \mathbf{I}_d \cos \varphi, \quad \text{if } \varphi \leq \pi/2, \quad (Eq.5.19)$$

$$= k_d \mathbf{M}_d \mathbf{I}_d \max(\mathbf{l} \cdot \mathbf{n}, 0) \quad (Eq.5.20)$$

$$\mathbf{I}_{specular} = k_s \mathbf{M}_s \mathbf{I}_s \cos^n \rho, \quad \text{if } \rho \leq \pi/2, \quad (Eq.5.23)$$

$$= k_s \mathbf{M}_s \mathbf{I}_s (\mathbf{r} \cdot \mathbf{v})^n, \quad \text{if } \rho \leq \pi/2, \quad (Eq.5.24)$$

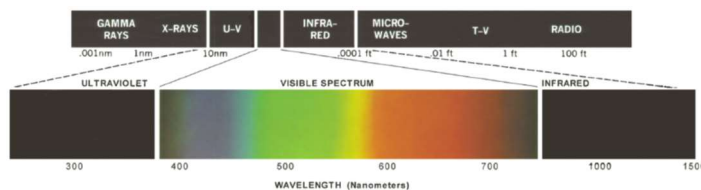
- Both the light  $\mathbf{I}$  and material  $\mathbf{M}$  contain RGB-information. Note that the are multiplied.



p. 30

## The RGB color model

- To perform correct calculations with color, the full spectrum with individual wavelengths should be regarded.



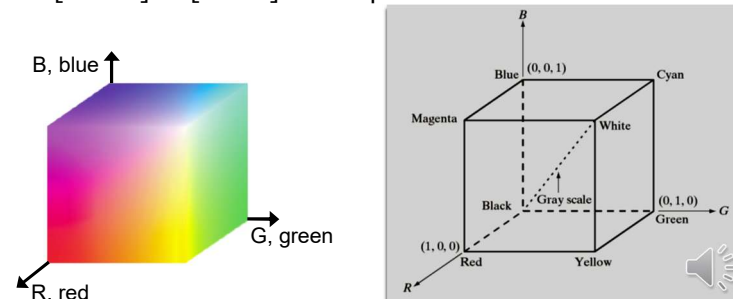
- The RGB color model is a simplified, approximate model that often works well, see next slide.



p. 31

## The RGB color model

- [R G B] can be regarded as an orthogonal coordinate system. It assumes additive color mixing.
- [R G B] = [1 1 1] corresponds to white.
- [R G B] = [0 0 0] corresponds to black.



p. 32



Assume white light:  $[1 \ 1 \ 1]$

p. 33

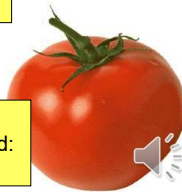
## Color of objects

An object that reflects light in all wave lengths (all RGB) appears white:  
 $[1 \ 1 \ 1] \cdot [1 \ 1 \ 1] = [1 \ 1 \ 1]$



An object that reflects blue light and absorbs green-red light appears blue:  
 $[1 \ 1 \ 1] \cdot [0 \ 0 \ 1] = [0 \ 0 \ 1]$

An object that reflects red light and absorbs blue-green light appears red:  
 $[1 \ 1 \ 1] \cdot [1 \ 0 \ 0] = [1 \ 0 \ 0]$



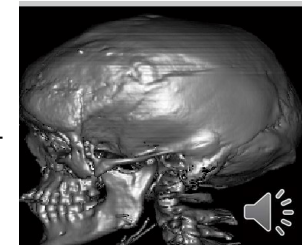
## Rotation

p. 34

■ To obtain the impression of rotation, do like this:

- Compute a sequence of projection images with different angles and angular difference  $> \approx 1^\circ$ .
- Show the images on the screen, one at a time in a suitable speed.
- Our brain gets the impression of a rotating object volume.

The film shows 46 views with  $4^\circ$  angular difference



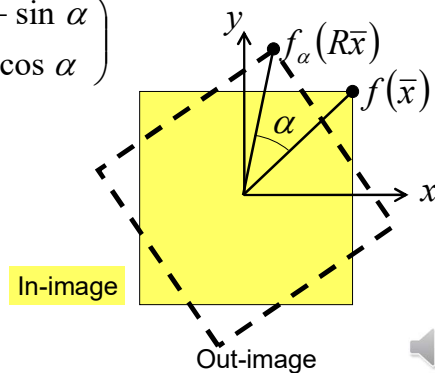
## Rotation in 2D, repetition

(e.g. from TSBB08, TSBB31)

p. 35

$$R = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

$$\bar{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$



## Rotation in 2D, repetition Inverse mapping

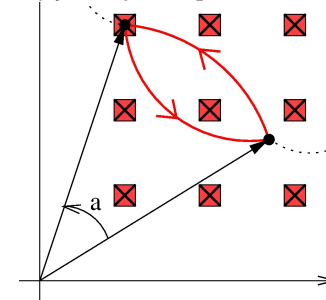
p. 36

- 1)  $(x, y)$  is given
- 3)  $f(x', y')$  is placed in  $(x, y)$

Observe!  
The in-image ■ and the out-image ✕ are overlaid.

- 2)  $(x', y')$  is calculated,  $f(x', y')$  is interpolated

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = R^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

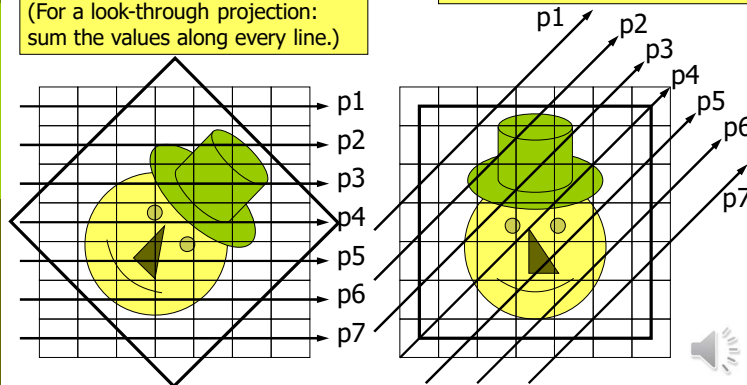


## Calculation of a rotated projection

p. 37

Alternative 1: Rotate every slice. Step forward along straight lines. (For a look-through projection: sum the values along every line.)

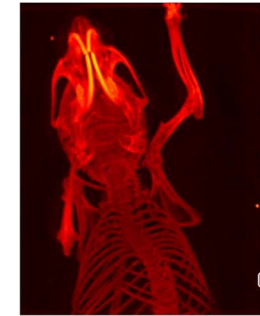
Alternative 2: Ray casting, see also slide p. 9. Step forward along slanted rays in the slice with  $\approx$  pixel distance. If necessary, interpolate the pixel values.



## MIP (Maximum intensity projection)

p. 38

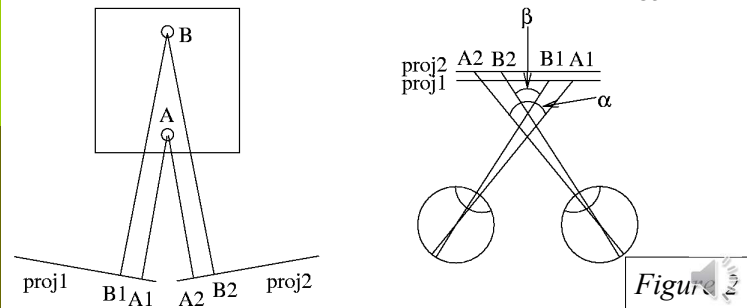
- Projects only the voxels with maximum intensity that fall in the way of parallel rays traced from the viewpoint to the plane of projection.
- To help the viewer's perception, rotation can be added to the MIP of the mouse here:
- Note that a MIP projection at angle  $0^\circ$  and angle  $180^\circ$  look the same, which can hinder the perception.
- An easy improvement to MIP is "Local maximum intensity projection". In this technique we don't take the global maximum value, but the first maximum value that is above a certain threshold.



## Computation of stereo images

p. 39

- Compute 2 projections with angular difference  $\approx 8^\circ$ .
- Show proj1 to the left eye and proj2 to the right eye.
- The brain gets the impression of looking at a 3D scene with two eyes. It matches  $A1-A2$  and  $B1-B2$ .  $A1-A2 = \alpha$ ,  $B1-B2 = \beta$ .  $\alpha > \beta \Rightarrow$  the brain believes that A is nearest.

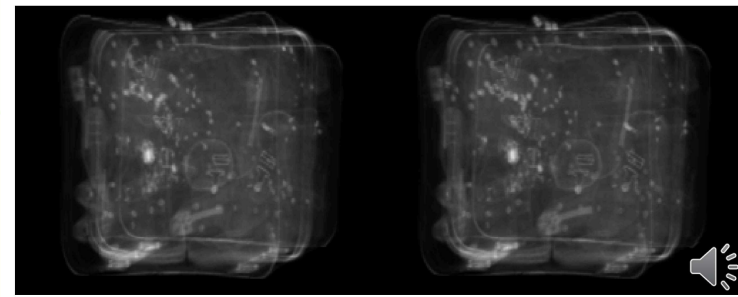


## A stereo pair of X-ray projections of a bag

p. 40

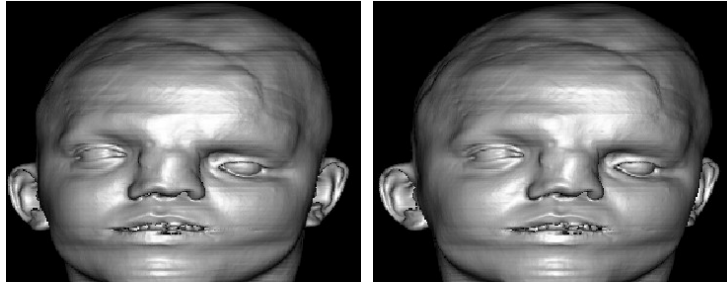
Figure 3

- Look at the figure with crossed gaze. It may be helpful to hold your finger between your eyes and the stereo pair.
- After a while, you can see three blurred images.
- Take away the finger, concentrate on the middle image and try to make it sharp and steady.



## A stereo pair of surface shaded images

p. 41



Lab!



## Stereo with red-green glasses

Lab!

Not 2020, though

p. 42



## 3D TV with active glasses

p. 43

**SA FUNGERAR 3D-TV**

Grundprincipen för 3D-tv med aktiva glasögon.

- 1 Det spelas in separata bilder för vänster och höger öga.
- 2 Bilder för vänster och höger öga spelas växelvis upp med 60 bildrutor per sekund (fps), vilket ger totalt 120 fps.
- 3 Glasögonen får signaler från en sändare som sitter på skärmen.
- 4 Signalen sluter växelvis vänster respektive höger glasögonlins, samtidigt som bilderna växlar på skärmen. Detta ger en tredimensionell upplevelse.

**Aktiva glasögon**

- Visar en bild i taget för ögonen.
- Passar många användare.

KÄLLA: Panasonic; REUTERS SVENSKA GRAFIKBYRÅN/TT

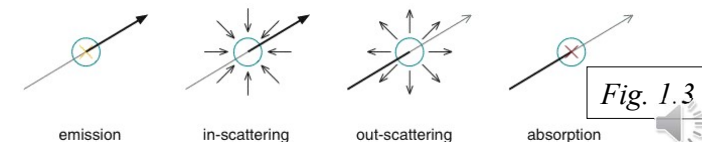
Östgöta Correspondenten • Lördag 9 januari 2010 A17



## Physical model of light transport

p. 44

- Emission (see next slide)
- Absorption (see next slide)
- Scattering
  - Light can be scattered by participating media, essentially changing the direction of light propagation. If the wavelength is not changed by scattering, the process is called elastic scattering. Conversely, inelastic scattering affects the wavelength.



## Emission and absorption (Scattering is now not considered.)

p. 45

### □ Emission

- The gaseous material emits light, increasing the radiative energy. In reality, for example, hot gas emits light by converting heat into radiative energy.

### □ Absorption

- Material can absorb light by converting radiative energy into heat. In this way, light energy is reduced.

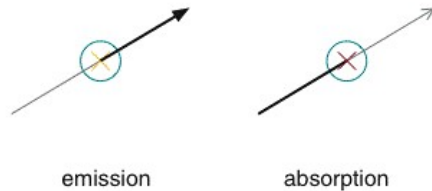


Fig. 1.3

## Ex) with emission/absorption, yellow bone and pink skin

p. 46

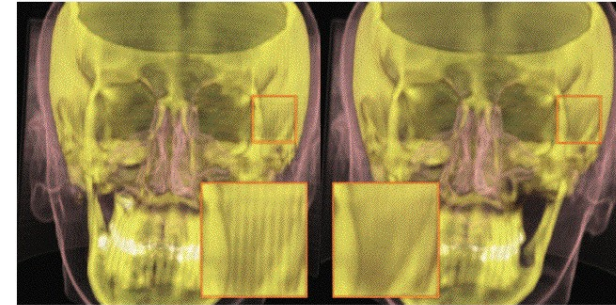


Figure 1.10. Comparison between trilinear filtering (left) and cubic B-spline filtering (right).

## Ex) An advanced Visualization of functional MRI (IMT, LiU)

p. 47

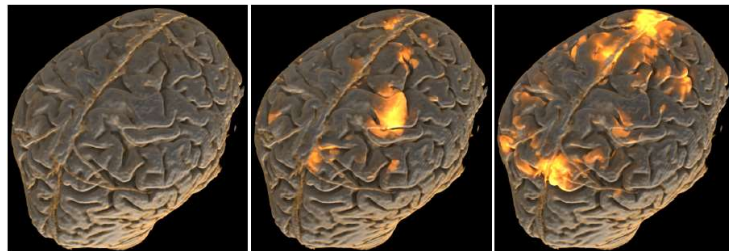


Figure 4: *Left:* The anatomy is rendered using the local ambient occlusion shading model, the  $A^L$  reflective light, which enhances the perception of depth. A diffuse shading component is also applied. *Middle:* The fMRI signal rendered using the LAO emission, i.e. including  $A^E$ . The image represents the brain activation during repeated motion of the left foot. *Right:* The activation during mathematical problem solving. The instructions, as well as the visualization of the brain activity as shown in these images, are shown to the subject in a head mounted display, and thus there is significant activation of the visual cortex.

## Ex) An advanced Visualization of functional MRI (IMT, LiU). Details.

p. 48

- A high resolution volume of the brain is measured with an MRI-scanner.
- This volume is visualized by diffuse surface shading.
- A low resolution 4D time sequence of the brain is registered and correlated with some activity, e.g. motion of the left foot. Activated brain area = increased blood flow.
- The low resolution detected activity is visualized as a light emitting area in the brain.



## Related to absorption: Beer's Law (or Beer-Lambert Law)

p. 49

- Intensity:  $I$
- Linear attenuation coefficient:  $\mu$  [1/m]

Diagram illustrating the differential change in intensity  $I$  as it passes through a small thickness  $\Delta y$  of a medium with attenuation coefficient  $\mu$ :

$$I \rightarrow \underbrace{\mu}_{\Delta y} \rightarrow I + \Delta I$$

$$\Delta I = -I \cdot \mu \cdot \Delta y \Rightarrow \text{differentiate}$$

$$dI = -I \cdot \mu \cdot dy$$

Diagram showing the integration of the differential equation over a path length  $L$  from  $I_{in}$  to  $I_{out}$ :

$$\Rightarrow \int_{I_{in}}^{I_{out}} \frac{dI}{I} = \int_0^L -\mu dy \Rightarrow [\ln I]_{I_{in}}^{I_{out}} = [-\mu \cdot y]_0^L \Rightarrow$$

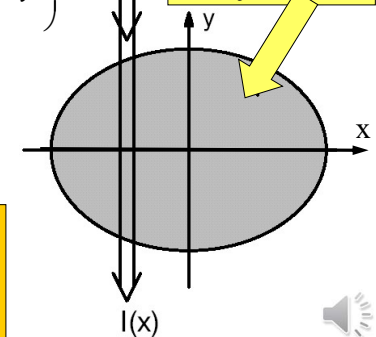
$$\ln I_{out} - \ln I_{in} = -\mu L \Rightarrow I_{out} = I_{in} e^{-\mu L}$$

## The attenuation of light or X-rays in an object

p. 50

$$I(x) = I_0 \exp \left( - \int_{-\infty}^{\infty} \mu(x, y) dy \right)$$

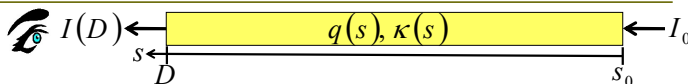
The attenuation function  $\mu(x, y)$  of the object



Compare with the  
"Volume-Rendering Integral"  
Eq.(1.7) on the next slide.

## Volume rendering integral

p. 51



### Volume-Rendering Integral

The emission-absorption optical model leads to the volume-rendering integral:

$$I(D) = I_0 e^{-\int_{s_0}^D \kappa(t) dt} + \int_{s_0}^D q(s) e^{-\int_s^D \kappa(t) dt} ds, \quad (1.7)$$

with optical properties  $\kappa$  (absorption coefficient) and  $q$  (source term describing emission) and integration from entry point into the volume,  $s = s_0$ , to the exit point toward the camera,  $s = D$ .

Radiance leaving the volume

Light entering from the background

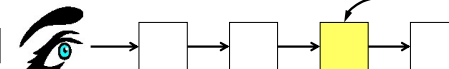
## Using Compositing to compute the volume rendering integral

p. 52

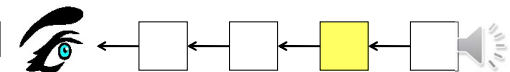
- Each pixel gives an intensity  $C_i$ , possibly in color, and an opacity value  $\alpha_i$ .
- With this technique, voxels with high  $\alpha_i$ -value can dominate, shadow, behind-lying pixels and light through forward-lying pixels.
- Composition can be performed Front-to-back or Back-to-front.
- I cannot follow the description in the book. Does it contain errors? On the next pages, I give you an alternative description.

$C_i$   
 $\alpha_i$

Front-to-back:



Back-to-front:



## Back-to-front compositing

- Assume that each sample on a view ray has color and opacity:

$$(C_0, \alpha_0), \dots, (C_b, \alpha_b), \quad C_i \in [0,1]^3, \alpha_i \in [0,1]$$

where the 0<sup>th</sup> sample is next to the camera

and the b<sup>th</sup> one is a (fully opaque) background sample:

$$C_b = (r, g, b)_{\text{background}} \quad \alpha_b = 1$$

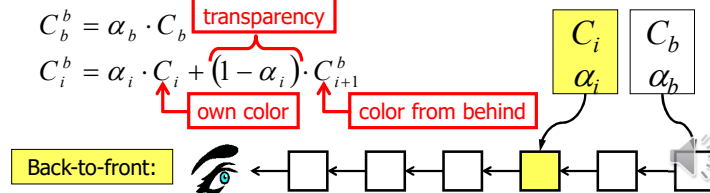
- Compositing can be defined recursively:

Let  $C_f^b$  denote the **composite color** of samples  $f, f+1, \dots, b$

Recursion formula for **back-to-front** compositing:

$$C_b^b = \alpha_b \cdot C_b$$

$$C_i^b = \alpha_i \cdot C_i + (1 - \alpha_i) \cdot C_{i+1}^b$$



## Back-to-front compositing

- The first few generations, written with **transparency**  $T_i = 1 - \alpha_i$

$$C_b^b = \alpha_b C_b$$

$$C_{b-1}^b = \alpha_{b-1} C_{b-1} + \alpha_b C_b T_{b-1}$$

$$C_{b-2}^b = \alpha_{b-2} C_{b-2} + \alpha_{b-1} C_{b-1} T_{b-2} + \alpha_b C_b T_{b-1} T_{b-2}$$

$$C_{b-3}^b = \alpha_{b-3} C_{b-3} + \alpha_{b-2} C_{b-2} T_{b-3} + \alpha_{b-1} C_{b-1} T_{b-2} T_{b-3} + \alpha_b C_b T_{b-1} T_{b-2} T_{b-3}$$

reveal the **closed formula** for compositing:

$$C_f^b = \sum_{i=f}^b \left( \alpha_i C_i \prod_{j=f}^{i-1} T_j \right)$$



## Front-to-back compositing

- Front-to-back** compositing can be derived from the closed formula: Let  $T_f^b$  denote the **composite transparency** of samples  $f, f+1, \dots, b$

$$T_f^b = \prod_{j=f}^b T_j$$

- Then the **simultaneous recursion** for front-to-back compositing is:

$$T_f^f = 1 - \alpha_f$$

$$C_f^{b+1} = C_f^b + \alpha_{b+1} C_{b+1} T_f^b$$

$$T_f^{b+1} = (1 - \alpha_{b+1}) T_f^b$$

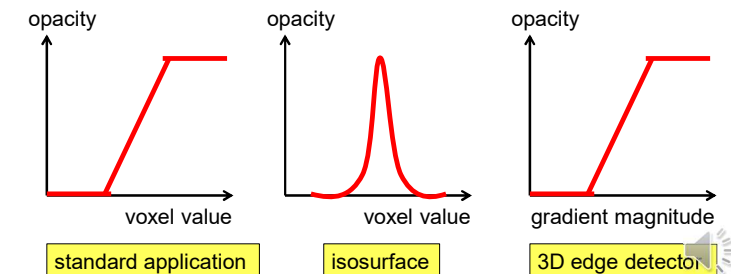
- Advantage of front-to-back compositing: **early ray termination** when composite transparency falls below a threshold



## Transfer functions, opacity

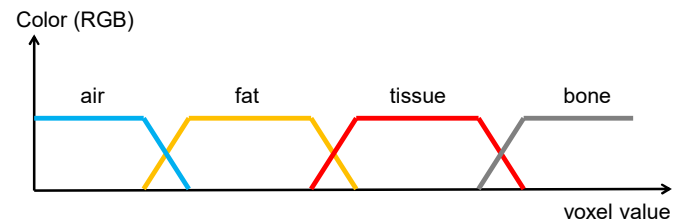
- By choosing different opacity transfer functions different types of applications can be achieved.

- Examples:



## Transfer functions, color

- The color transfer function allows to make a simple **classification**.
- Example appropriate for a CT-volume:



- Note: Better (but more expensive) classification can be obtained by segmentation.



## Volume Ray Casting with compositing and shading

- 1. Ray casting
- 2. Sampling (and interpolation)
- 3. Shading
- 4. Compositing
- [http://en.wikipedia.org/wiki/Volume\\_ray\\_casting](http://en.wikipedia.org/wiki/Volume_ray_casting)

