

# Guide to answers for written examination in TSBB09 Image Sensors, 2017-01-09

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## PART I: STANDARD CAMERAS & IR SENSORS

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**Exercise 1** See lecture A, slides 35–.

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**Exercise 2** See lecture A, slides 73–.

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**Exercise 3** The camera will detect infrared radiation that is emitted also from the media between the object and the camera (for example: air), from the lens of the camera, and from other parts of the camera itself. In addition, the surface of the object can reflect infrared radiation from other objects, which also distorts the temperature measurement. This was discussed in one the questions of Computer Exercise B.

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**Exercise 4** See lecture B, slides 67–.

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**Exercise 5** The variance depends linearly on the image intensity,  
 $\text{imvar} = \text{const} \cdot \text{imaver}$ .

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**Exercise 6** In very bright areas, the sensor is saturated to the highest possible value and consequently the variance becomes zero.

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**Exercise 7** Filter wheel: Advantage: measures a 2D image in one "shot". Disadvantage: Can only measure a smaller number of wavelength bands, corresponding to a filter. Lower temporal resolution since the filter wheel must be able to rotate a full turn to produce all wavelength bands. Beam-splitter: Advantage: simple to measure a large number of wavelength bands with high resolution. High temporal resolution. Disadvantage: measures several wavelength bands but only a long a line. To get an image, the sensor must scan over the scene.

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**Exercise 8** See lecture B, 60. Examples of distortion caused by lenses and a wide aperture are: blurring of the scene outside the object plane (73-), geometric distortion (78-), vignetting (83-), chromatic aberration (88-). There is also diffraction (64-), but this effect happens in a pinhole camera too.

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## PART II: GEOMETRY AND MULTIPLE VIEWS

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**Exercise 9** See IREG, Section 8.2.2.

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**Exercise 10** If the two projection lines intersect, this is the same as to say that the two image points can be corresponding points, i.e., it is possible that they correspond to one and the same 3D point. If the two projection points don't intersect, they two image points cannot correspond to the same 3D point. Consequently, the question of whether the projection lines intersect or not is same question as whether the two image points (the projections) can be corresponding points or not. This can be tested using the epipolar constraint, based on the fundamental matrix  $\mathbf{F}$ .

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**Exercise 11** The *outer parameters* are in  $[\mathbf{R} \ \mathbf{t}]$ . They depend on how the camera is *oriented* in relation to the world coordinate system.

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**Exercise 12**

$$\mathbf{A} = \begin{pmatrix} 1500 & 0 & 499.5 \\ 0 & 1470 & 299.5 \\ 0 & 0 & 1 \end{pmatrix}$$

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**Exercise 13** It is a model of the lens distortion.

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**Exercise 14** The camera calibration matrix. (It was denoted  $\mathbf{K}$  in the panorama computer exercise.)

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**Exercise 15**

- a) Corresponding points in *a) pattern* and *b) calibration image* should be chosen. Order is important, but not rotation. One solution is to let  $(X_1, Y_1) = (0, 0)$ ,  $(X_2, Y_2) = (5, 0)$ ,  $(X_3, Y_3) = (10, 0)$  correspond to  $(u_1, v_1) = (86, 26)$ ,  $(u_2, v_2) = (165, 37)$ ,  $(u_3, v_3) = (243, 48)$ .

- b)
- Determine corresponding points  $(X_i, Y_i)$ ,  $i = 1, \dots, N$  and  $(u_i, v_i)$ ,  $i = 1, \dots, N$ .
  - Denote equation (3) as  $\mathbf{D} \cdot \mathbf{h} = \mathbf{f}$  and solve  $\mathbf{h}$  by  $\mathbf{h} = \mathbf{D}^+ \cdot \mathbf{f}$ .
  - Reshape  $\mathbf{h}$  to  $\mathbf{H}$ .
  - Measure the object endpoints  $(u_a, v_a)$  and  $(u_b, v_b)$  in pixels in *c) object image*.
  - Compute corresponding real world coordinates  $(X_a, Y_a)$  and  $(X_b, Y_b)$  in centimeters by

$$s_a(X_a, Y_a, 1)^T = \mathbf{H}^{-1} (u_a, v_a, 1)^T,$$

$$s_b(X_b, Y_b, 1)^T = \mathbf{H}^{-1} (u_b, v_b, 1)^T.$$

- The length of the object is finally given by  $\sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2}$ .

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**Exercise 16** If  $\det \mathbf{F} \neq 0$  it means that the epipolar points are not well-defined, since they are left and right null vectors of  $\mathbf{F}$ . Normally, the epipolar lines must intersect at the epipolar points in each image, but when the epipolar points are not well-defined, the epipolar lines will no longer intersect at a single point in each image. See IREG, Section 14.2.1, under “constraint enforcement”.

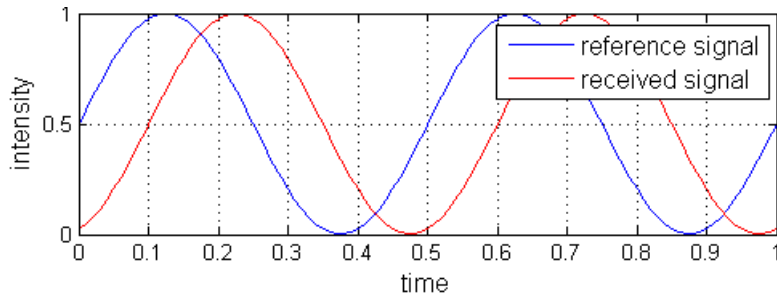
## PART III: NON-STANDARD IMAGE SENSORS

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**Exercise 17** One alternative is to use different exposure times for the different images. Another alternative is to use two (or more) light sensors for each pixel. One that has a higher sensitivity to light, for example by having a larger area, and one that has a lower sensitivity (smaller area). See lecture J, slides 2–.

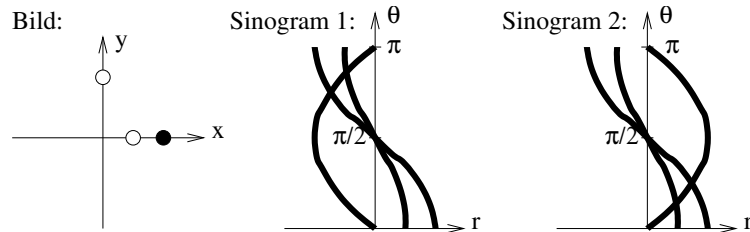
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**Exercise 18** See figure. The phase difference between the reference signal and the received signal gives the time difference, which gives the range. There is an ambiguity in phase/time difference. In the figure, time difference can be 0.1 or 0.6.



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**Exercise 19** Depending on the definition of the positive angle direction, there are two different solutions, see figure.



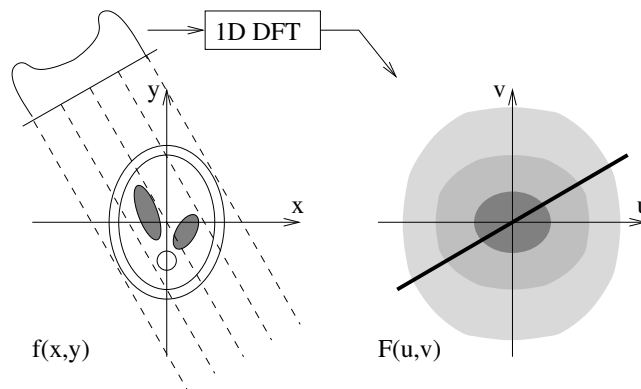

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**Exercise 20**

- The PI-method developed at LiU: **Approximate**
- WFBP (weighted filtered backprojection) from Siemens: **Approximate**
- Katsevich's algorithm: **Exact**

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**Exercise 21** The position of the Fourier transformed projection is marked with a thick line in the figure.




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**Exercise 22** The parameter  $n$  controls the shininess. A perfectly shiny surface has an infinitely large  $n$  and reflects the light in only one direction. In practice, a lower  $n$ , i.e.  $n=5$ , is suitable for a shiny surface. Then the light is reflected in a larger solid angle around the main direction.

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### Exercise 23

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for z=-127 to 128
  for x=-127 to 128
    D(x,z):=0;
    y:=-128;
    do
      y:=y+1;
      while (f(x,y,z)<77 and y<128)
        if (f(x,y,z)<77)
          D(x,z):=0;
        else
          D(x,z):=128-y;
        end
      end
      M(x,z):=0;
      for y=-127 to 128
        if (f(x,y,z)>M(x,z))
          M(x,z):=f(x,y,z);
        end
      end
    end
  end
end
end

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**Exercise 24** Due to the varying object reflectivity, the obtained laser intensity varies approximately as indicated in figure a). The laser intensity maximum gives the real position of the detected laser point and it is indicated with a white dot in a), and correspondingly in b) and c). Draw a line through the sensor position and the camera center. The cross-section of this line with the laser sheet is the range. Both the desired and the obtained position of the range are indicated in the figure.

