TSBB21, Lecture A
Image Formation
Part 1

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Basic physics

- Electromagnetic radiation consists of electromagnetic waves
  - They have energy
  - They propagate through space
- The waves consist of *transversal* electrical and magnetic fields that alternate with a temporal frequency ν (Hertz) and spatial wavelength λ (meter)

Frequency and wavelength

- The relation between frequency and wavelength is

$$\boxed{c = \lambda\nu}$$

c is the speed of light and depends on the medium, $c \leq c_0$

- c_0 = speed of light in vacuum ≈ $3 \cdot 10^8$ m/s

Particles and energy

- Light can also be represented as particles, *photons*
- The energy of a photon is

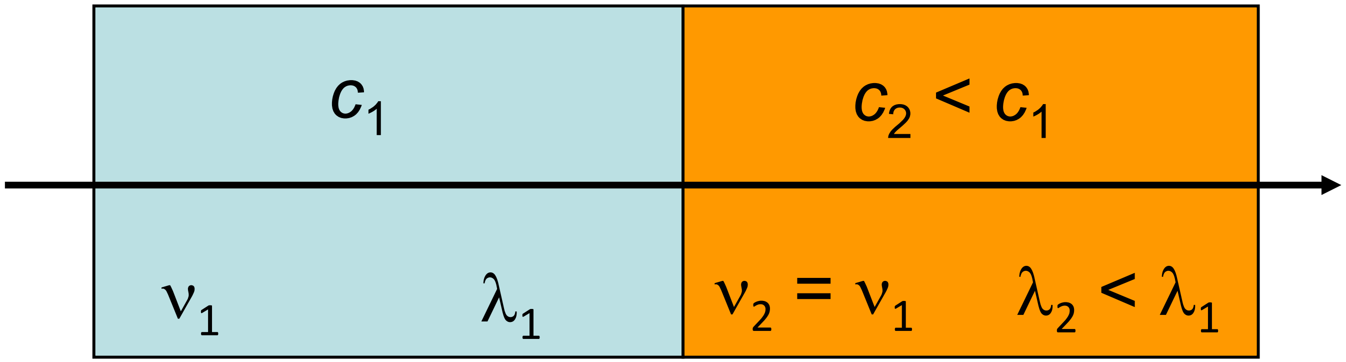
$$E = h\nu = hc/\lambda$$

Energy increases with ν and decreases with λ

h is Planck's constant ($\approx 6.623 \bullet 10^{-34}$ Js)

Particles and energy

- Energy depends on the frequency ν ($E = h\nu = h\,c\,/\,\lambda$)
- Energy is preserved when travelling from one medium to another



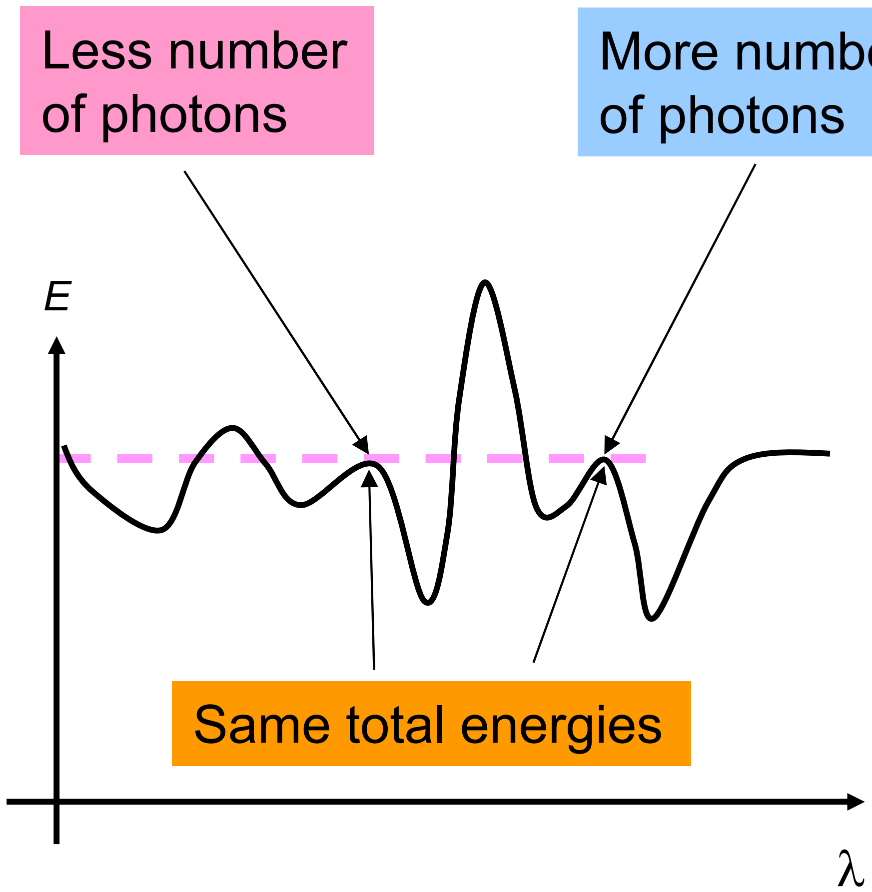
c_2 and λ_2 must change with the same factor relative to c_1 and λ_1

- If the speed of light changes from one medium to another,
  - the frequency ν is constant to make the energy constant
  - the wavelength λ must change

Spectrum

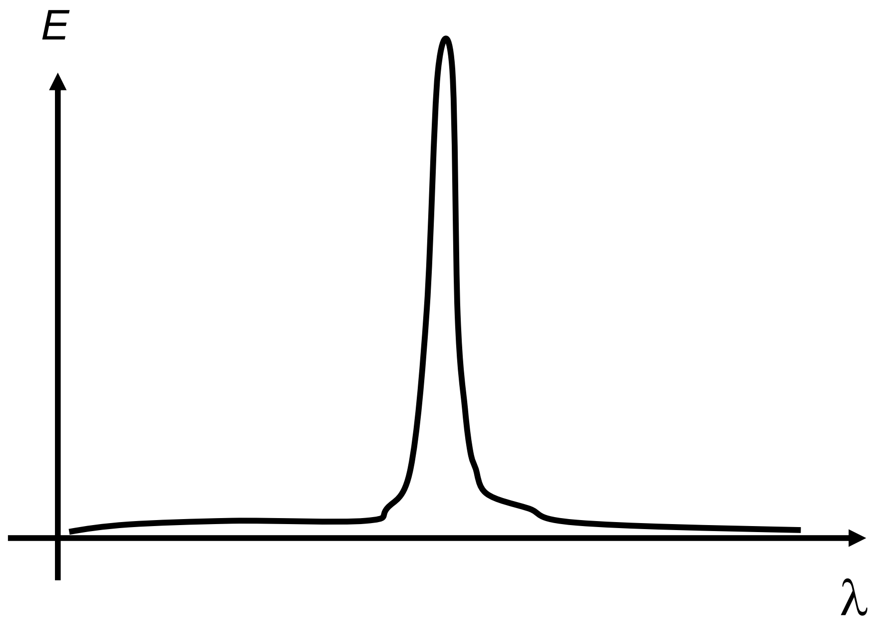
- In practice, light normally consists of
  - photons with a range of energies, or
  - waves with a range of frequencies
  - This mix of frequencies/wavelengths/energies is called the *spectrum* of the light
- The spectrum is a function that gives the *total amount* of energy for each frequency or wavelength
- Monochromatic light consists essentially of only one frequency/wavelength
  - Can be produced by special light sources, e.g., lasers

Spectrum



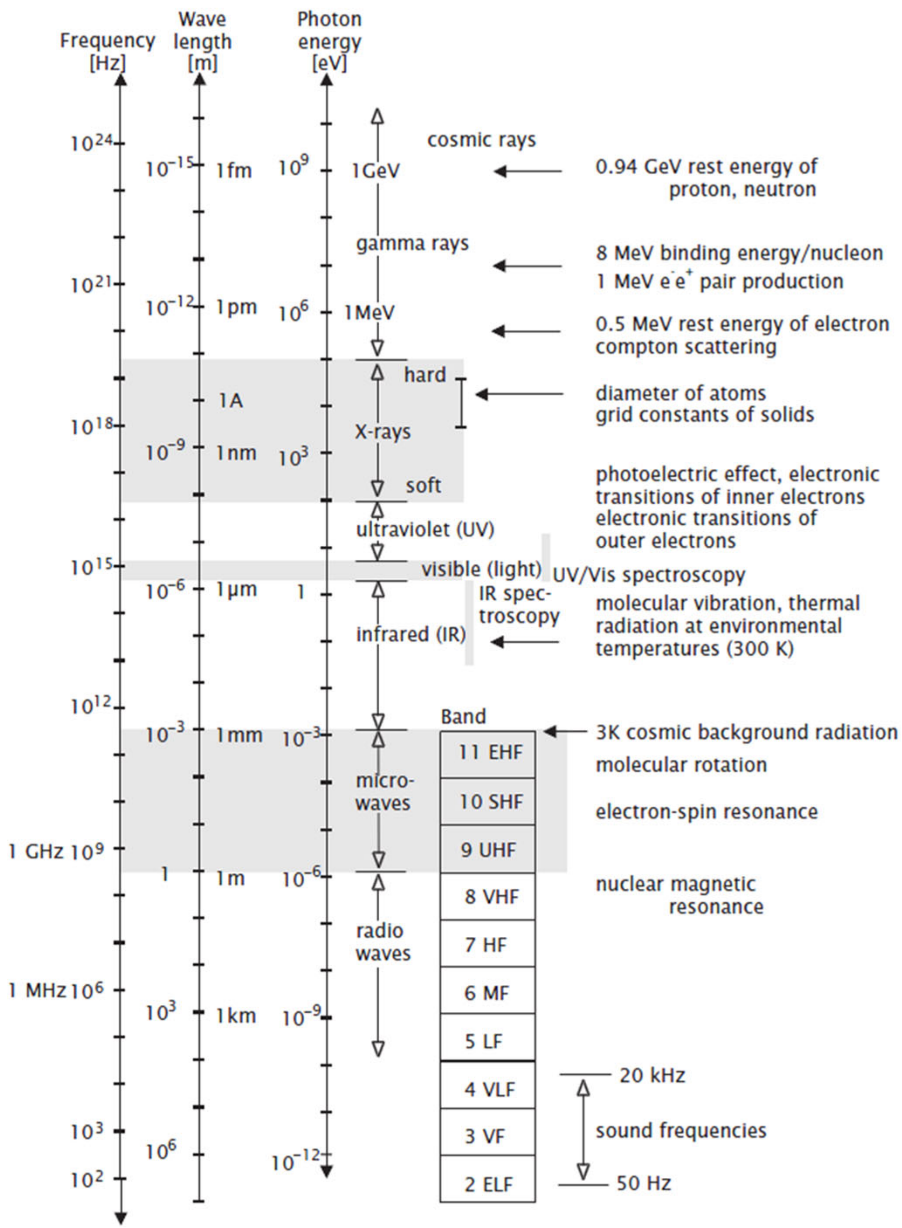
”Natural light”

$$E = h\,\nu = h\,c\,/\,\lambda$$



Monochromatic light

Classification of the Electromagnetic spectrum

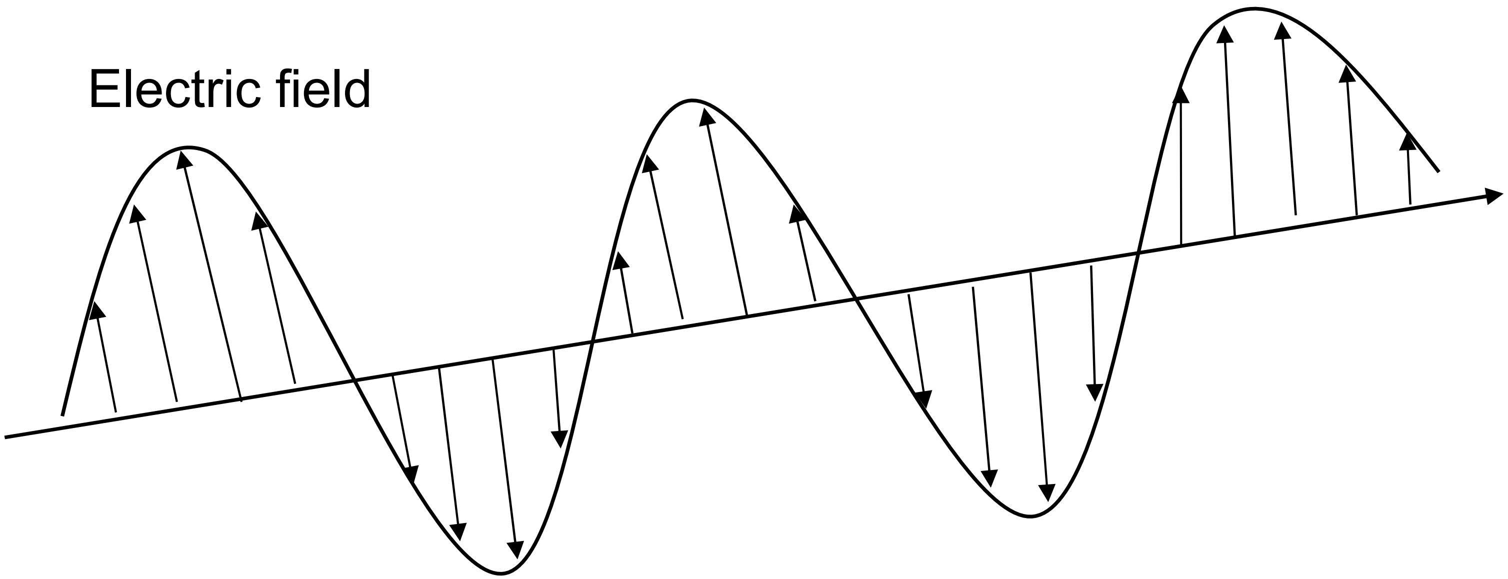


Polarization

- The electromagnetic field has a *direction*
  - Perpendicular to the direction of motion
- The *polarization* of the light is defined as the direction of the electric field
- Natural light is a mix waves with polarization in all possible directions: *unpolarized light*
- Special light sources or filters can produce *polarized light* of well-defined polarization

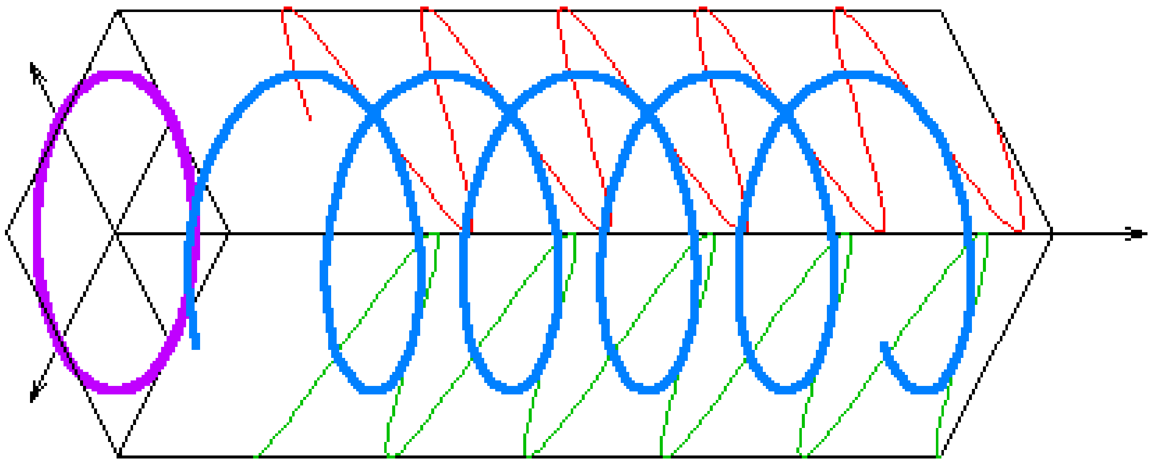
Polarization

- Plane polarization
  - The electric field varies only in a single plane



Polarization

- Circular/elliptical polarization
  - The electric field vector rotates
  - Can be constructed as the sum of two plane polarized waves with 90^o phase shift



- Conversely: plane polarized light can be decomposed as a sum of two circular polarized waves that rotate in opposite directions

Coherence

- The phase of the light waves can either be
  - random: *incoherent light* (natural light)
  - in a systematic relation: *coherent light*
- Coherent light is usually related to monochromatic light sources (e.g. laser)
- Compare a red light-emitting diode (LED) and a red laser
  - Both produce light within a narrow range
  - The red LED light is incoherent
  - The red laser light is coherent

Radiometry

- Light radiation has *energy*
  - Each photon has a particular energy related to its frequency ($E = h\,\nu$)
  - The number of photons of a particular frequency gives the amount of energy for this frequency
  - This is described by the spectrum
  - Unit: Joule (or Watt second)
  - Is usually not measured directly

Radiometry

- The power of the radiation, i.e., the energy per unit time, is the *radiant flux*
  - Since the energy depends on the frequency, so does the radiant flux
  - Unit: Watt or Joule per second
  - Is usually not measured directly

Radiometry

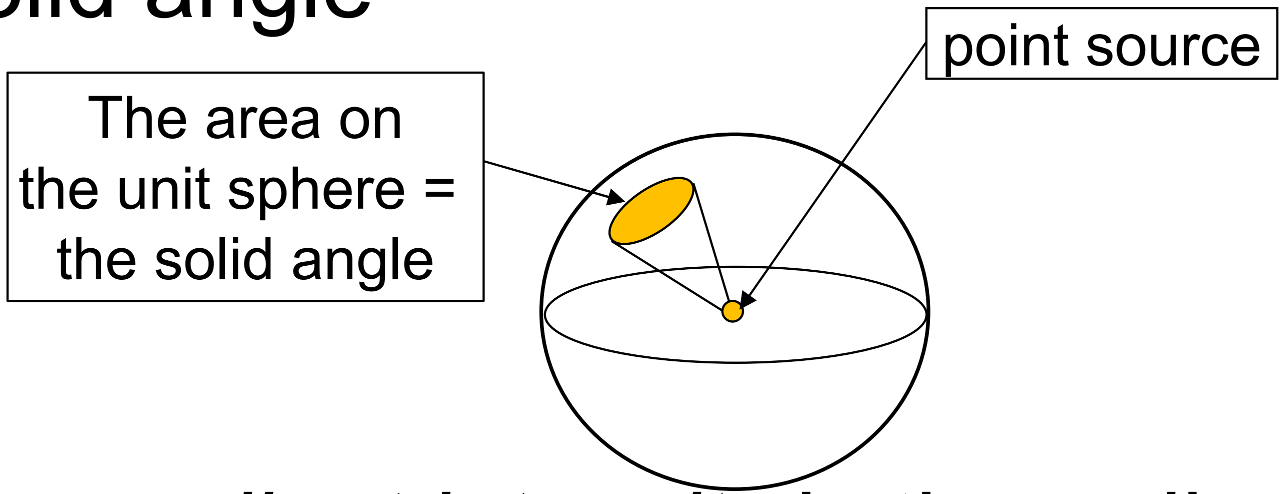
- The radiant flux per unit area is the *radiant flux density*
  - Since the flux depends on the frequency, so does the flux density
  - Unit: Watt per square meter
  - Can be measured directly!

As the energy through a specific area during a specific time interval

- *Irradiance*: flux density incident upon a surface
- *Excitance* or *emittance*: flux density emitted from a surface

Radiometry

- For point sources, or distant sources, the flux density can also be measured per unit solid angle

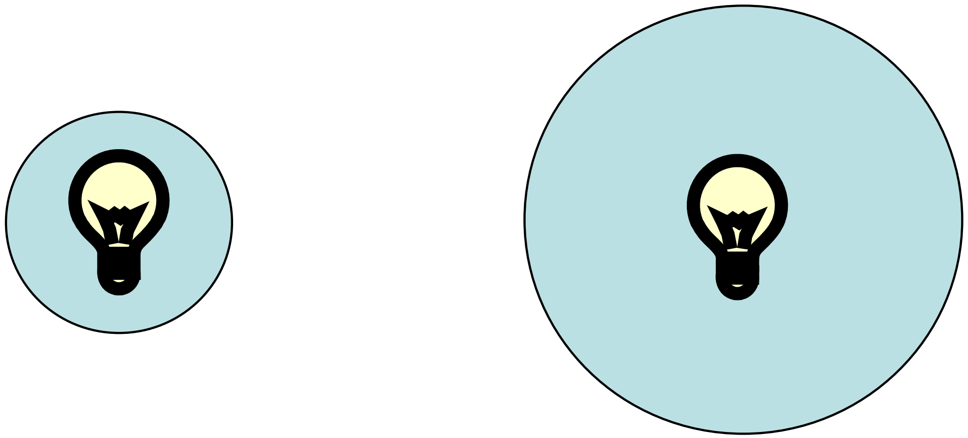


- The *radiant intensity* is the radiant flux per unit solid angle
  - Unit: Watt per steradian

    (the whole sphere corresponds to 4π steradians)

Basic principle

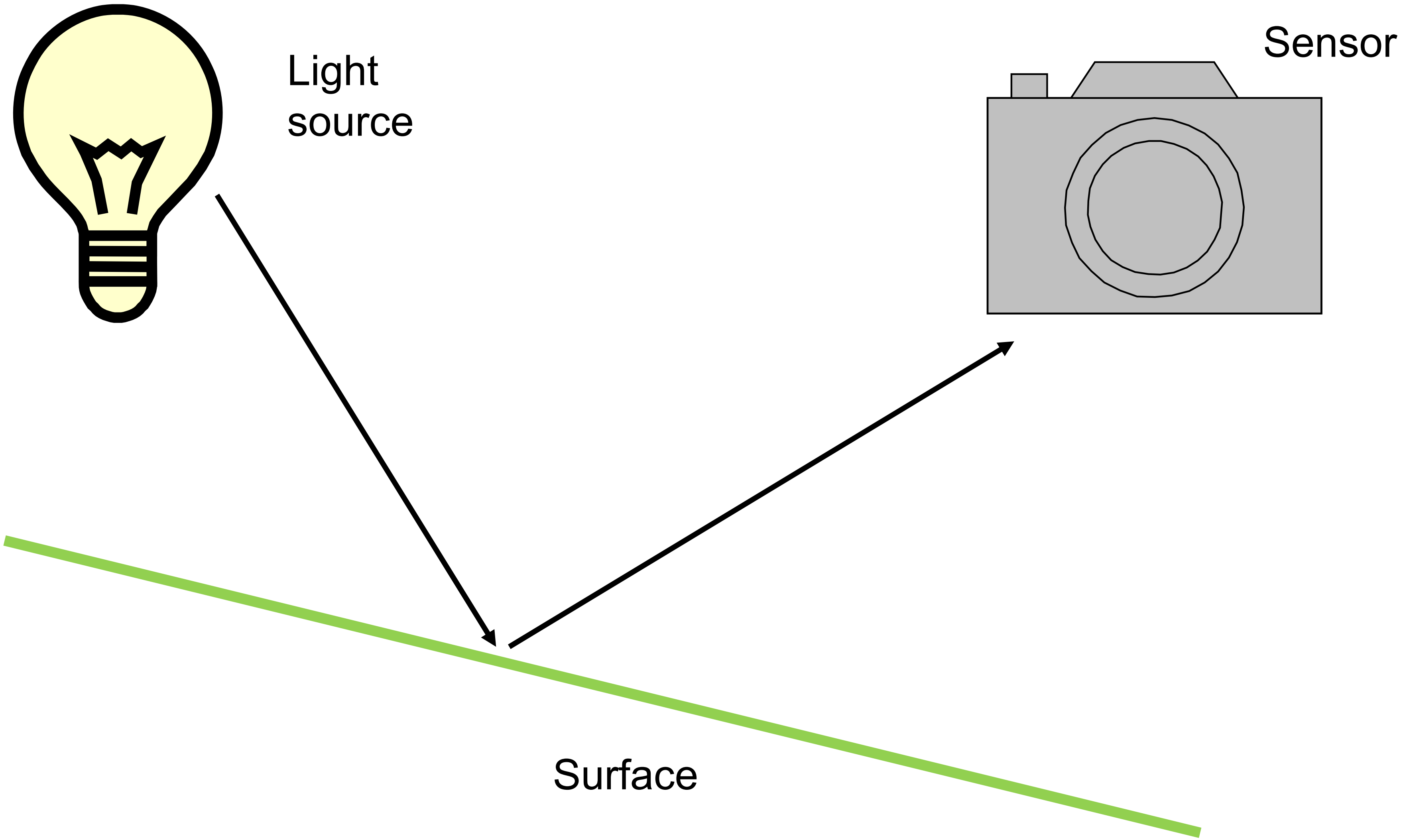
- Based on preservation of energy
  - A constant light source must produce the same amount of energy through a solid angle regardless of distance to the source
    - The *radiant intensity* is constant with respect to distance
    - The *radiant flux* density decreases with the square of the distance to the source



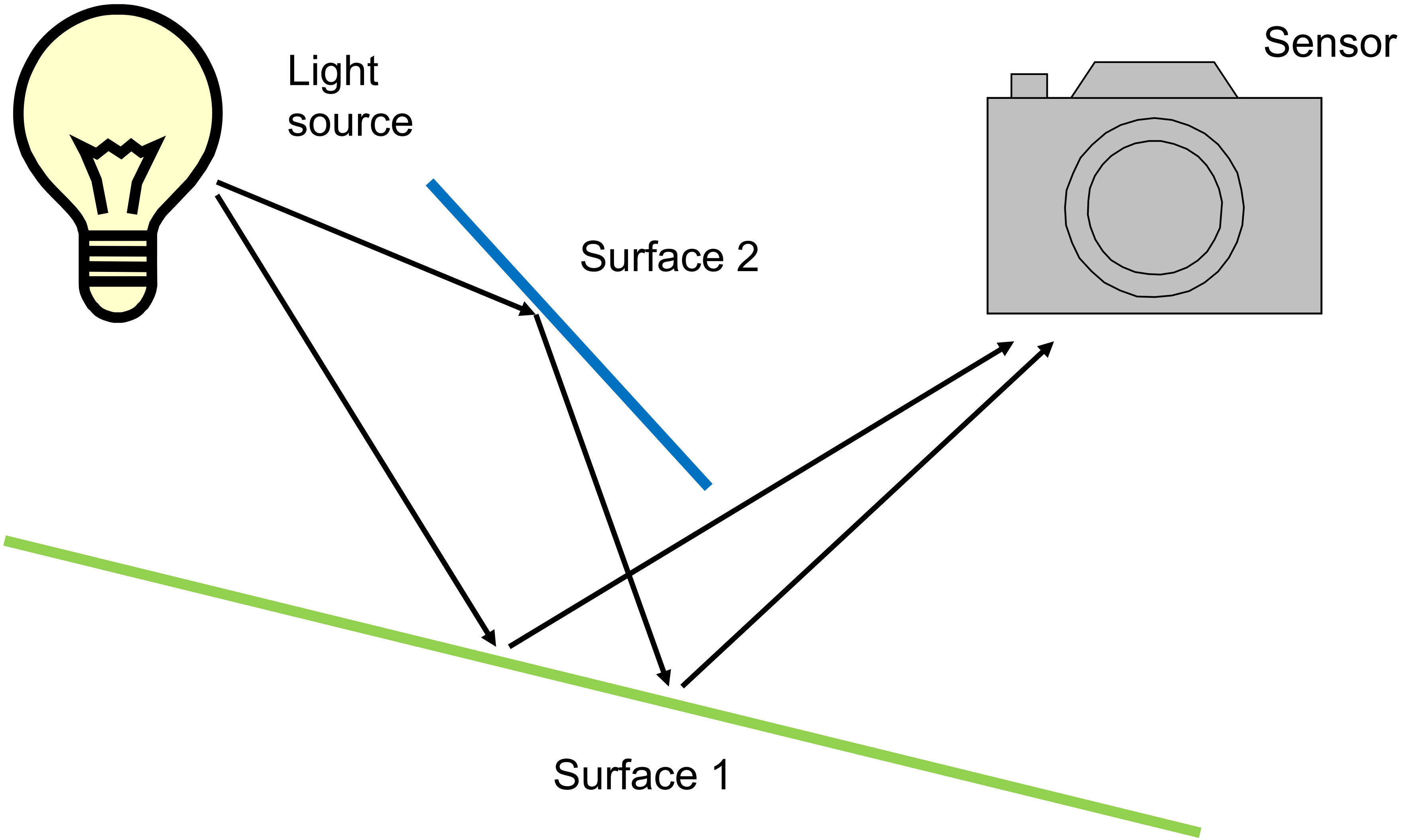
The radiometric chain

- By the radiometric chain; we mean the system consisting of:
  - light sources
  - objects that reflect the light
  - sensors that detect the light
  - medias which the light has to pass

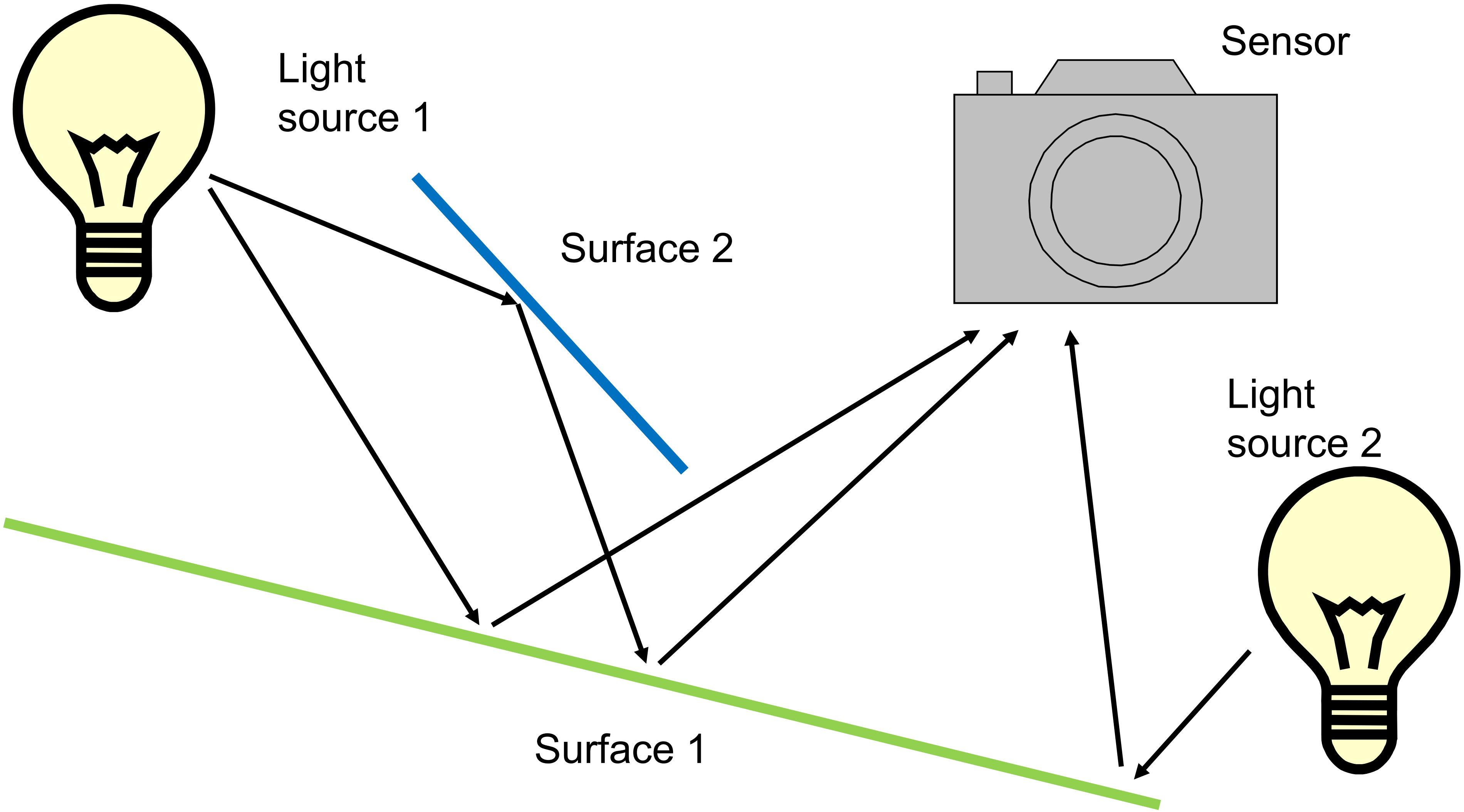
The radiometric chain



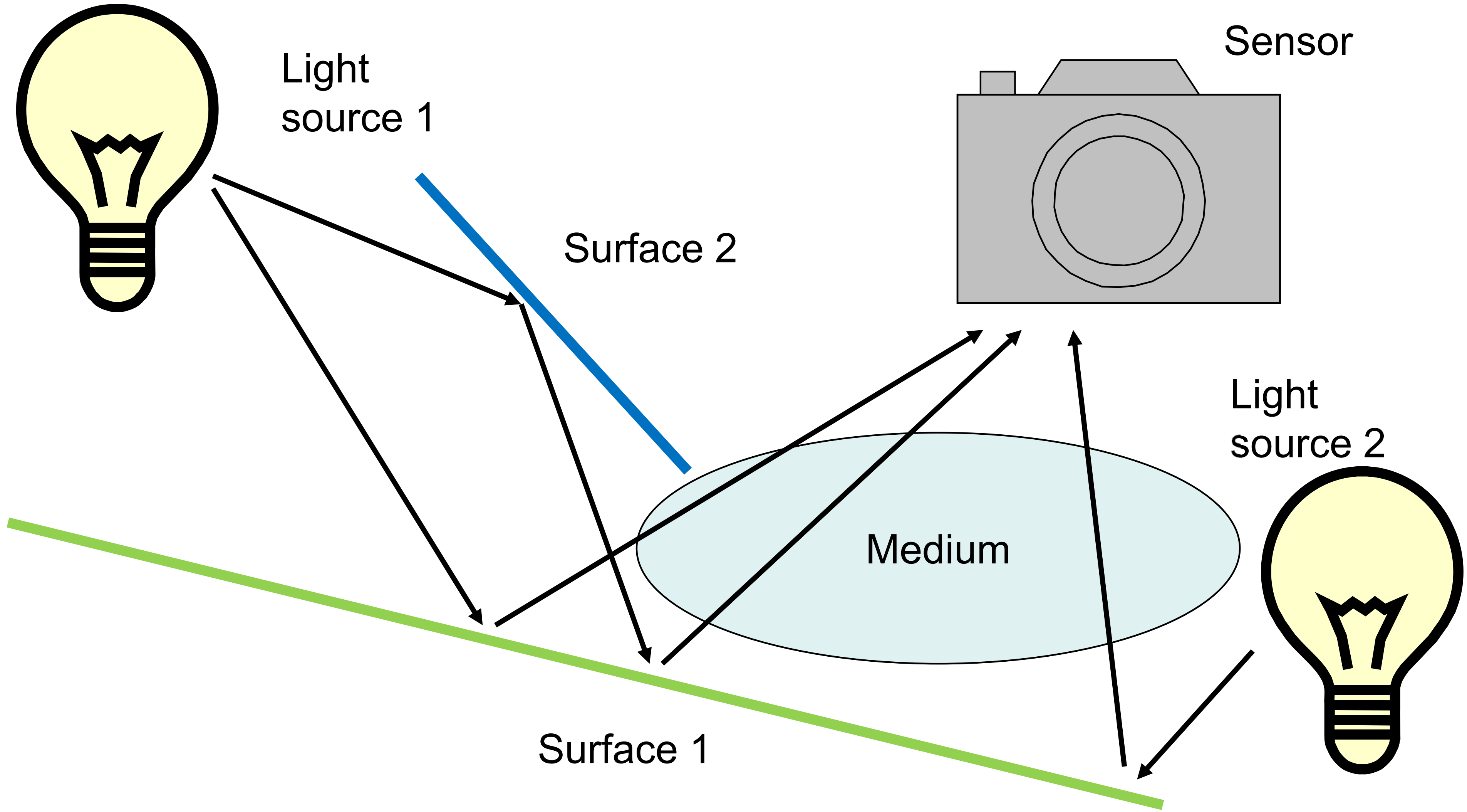
The radiometric chain



The radiometric chain



The radiometric chain



End of Part 1

Questions?

TSBB21, Lecture A
Image Formation
Part 2

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Interaction between light and matter

- Most types of light-matter interactions can be represented by

  n = the material's refractive index

  α = the material's absorption coefficient

- Both parameters depend on λ
- More complex interactions include polarization effects or non-linear effects

Light incident upon a surface

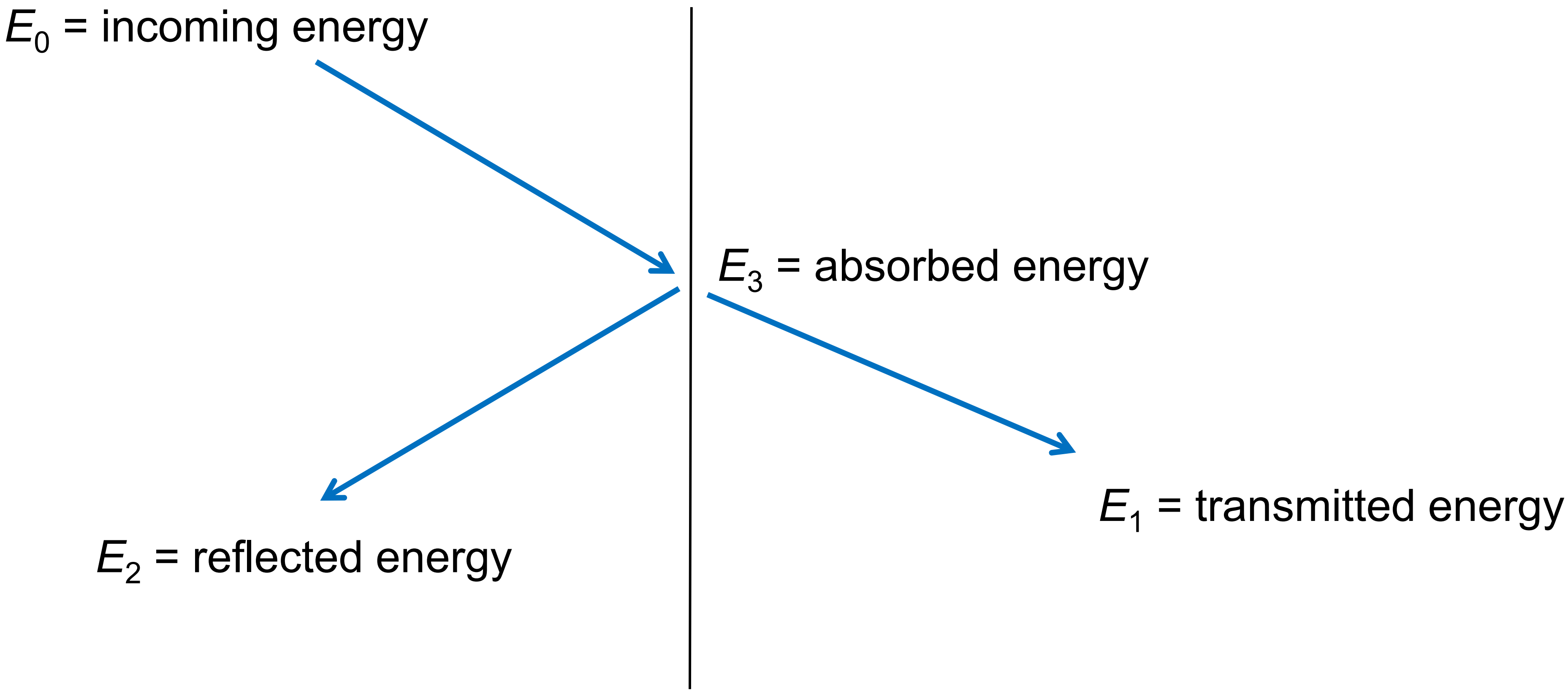
- When light meets a surface
  - Some part of it is transmitted through the new media
    - Possibly with another speed and direction
  - Some part of it is absorbed by the new media
    - Usually: the light energy is transformed to heat
  - Some part of it is reflected

- For the same material, all three effects depend on the light's wavelength λ
  - Equivalently: they depend on the light's frequency ν

Basic principle

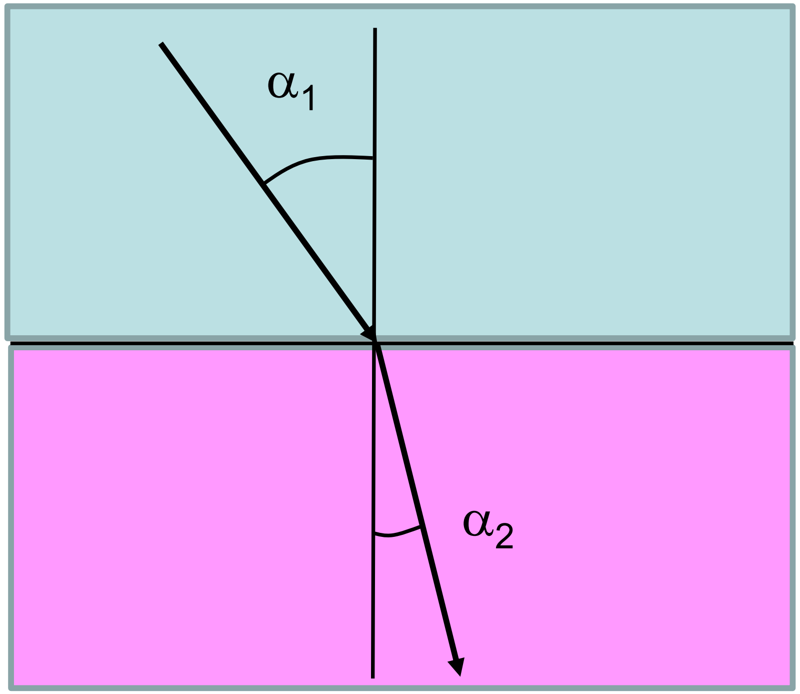
Based on preservation of energy:

$$E_0 = E_1 + E_2 + E_3$$



Refraction

- The light that is transmitted into the new medium is *refracted* due to the change in light speed



Snell's law of refraction:

$$\frac{\sin\alpha_2}{\sin\alpha_1} = \frac{n_1}{n_2} = \frac{c_2}{c_1}$$

Absorption

- Absorption implies attenuation of transmitted or reflected light
- Materials get their colors as a result of different amounts of absorption for different wavelengths
  - Ex: A red object attenuates wavelengths in the red band less than in other bands. These wavelengths are reflected (or transmitted) instead.

Absorption

- The absorption of light in matter depends on the length that the light travels through the material

$$\boxed{a = e^{-\alpha x}}$$

- a = attenuation of the light ($0 \leq a \leq 1$)
- α = the material's absorption coefficient
- x = length that the light travels in the material

Absorption spectrum

- The spectrum of the reflected/transmitted light is given by

$$\boxed{s_2(\nu) = s_1(\nu)a(\nu)}$$

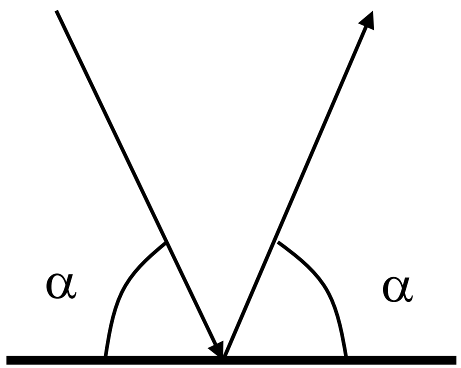
s_1 = incident spectrum

s_2 = reflected/transmitted spectrum

a = absorption spectrum ($0 \leq a(\nu) \leq 1$)

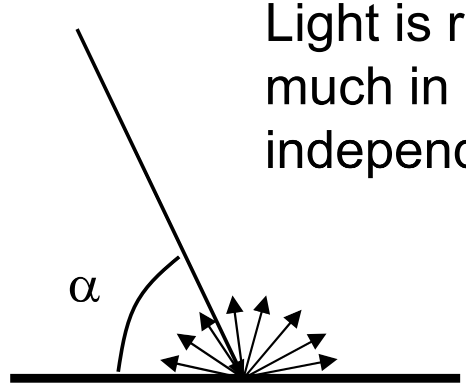
Surface reflection

- Highly dependent on the surface type



Mirror

Light is reflected equally much in all directions independent of α



Lambertian surface

A real surface is often a mix between the two cases

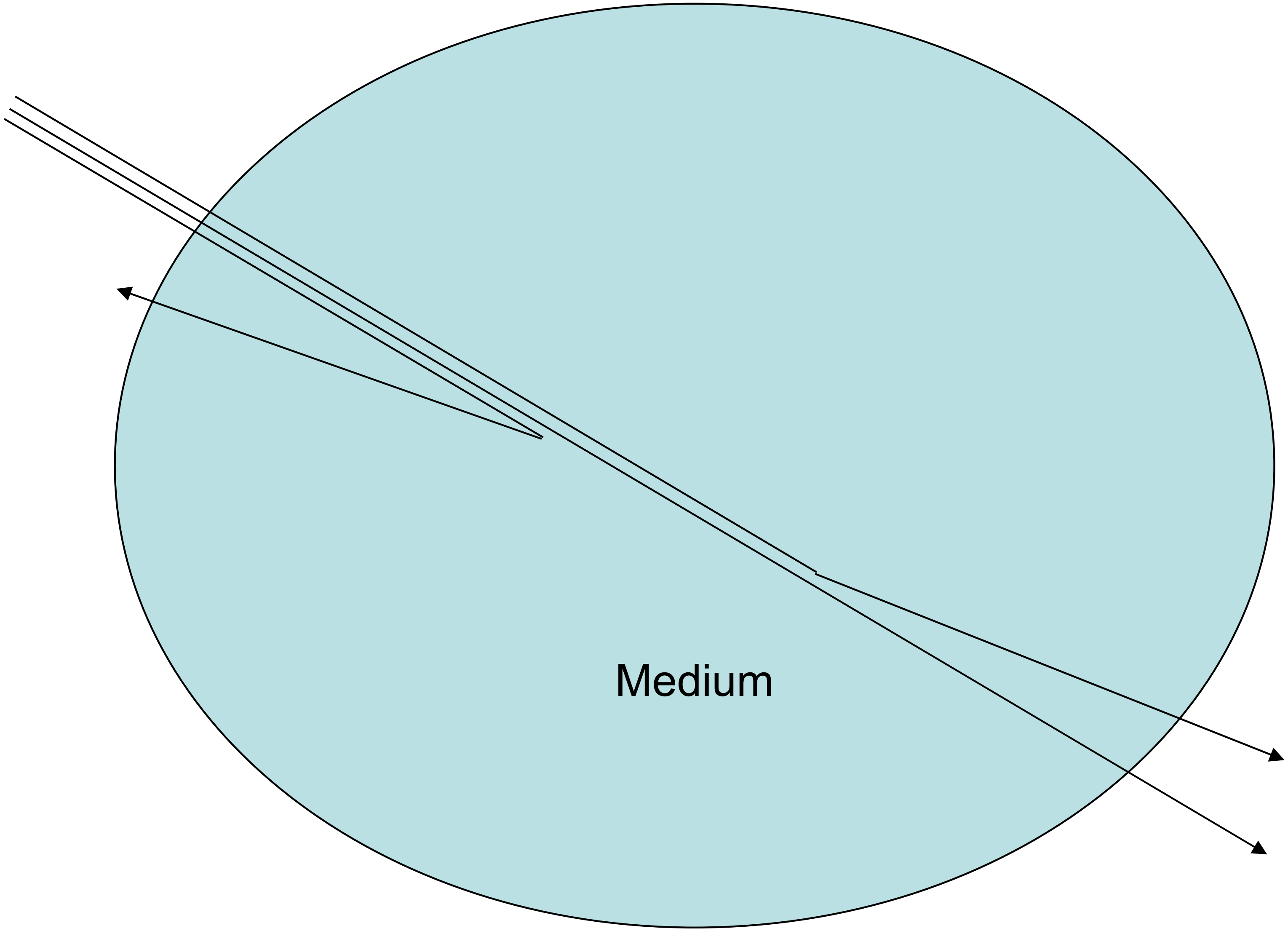
Emission

- Independent of its interaction with incident light (well, almost…):
  – Any object, even one that is not considered a light source, emits electromagnetic radiation
- Primarily in the IR-band, based on its temperature

Scattering

- All mediums (other than vacuum) *scatter* light
  - Examples: air, water, glass
- We can think of the medium as consisting of small particles and with some probability they reflect the light
  - In any possible direction
  - Different probability for different directions
  - Weak effect and roughly proportional to λ^{-4}
  - In general, the probability depends also on the distribution of particle sizes

Scattering

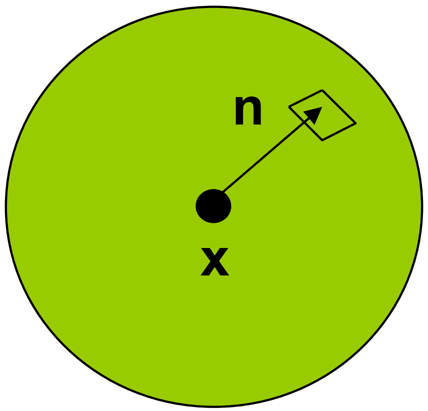


Scattering

- Scattering is not an absorption
- It rather means that the light ray does not travel along a straight line through the medium
  - There is a probability that a certain photon exits the medium in another direction than it entered.
- Examples:
  - The sky is blue because of scattering of the sun light
    - Blue light has shorter wave-length and is scattered more by particles in the atmosphere
  - A strong laser beam becomes visible in air

The plenoptic function

- At a point $\mathbf{x} = (x_1,x_2,x_3)$ in space we can measure how much light energy that travels in the direction $\mathbf{n} = (n_1,n_2,n_3)$, $||\mathbf{n}|| = 1$



The plenoptic function

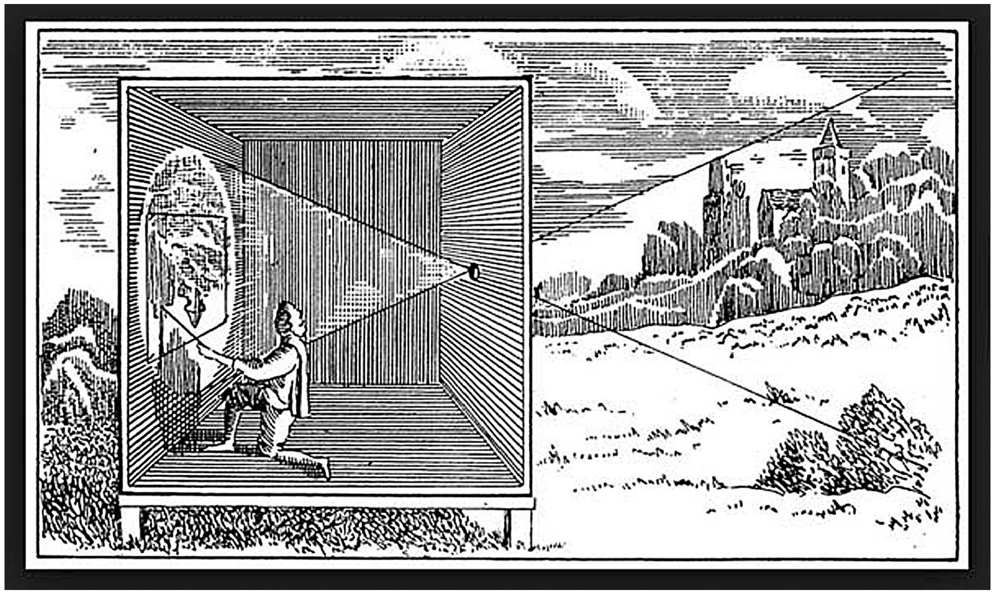
- The plenoptic function is the corresponding radiance intensity function
  - $p(\mathbf{x},\mathbf{n})$ (5-dim. since $\mathbf{x}$ is 3-dim. and $\mathbf{n}$ has 2 degrees of freedom.)
- Can also be a function of
  - Frequency ν
  - Time t
  - $p(\mathbf{x},\mathbf{n},\nu,t)$ (7-dim)
  - (Polarization)

A light camera

- A (light) camera is a device that *samples* the plenoptic function in a particular way
- Different types of cameras sample in different ways
  - Pinhole-camera
  - Orthographic camera
  - Push-broom camera
  - Light-field camera
  - …

The pinhole camera

- The most common camera model is the *pinhole camera*
  - Swedish: *hålkamera*
- An ideal model of the *camera obscura*

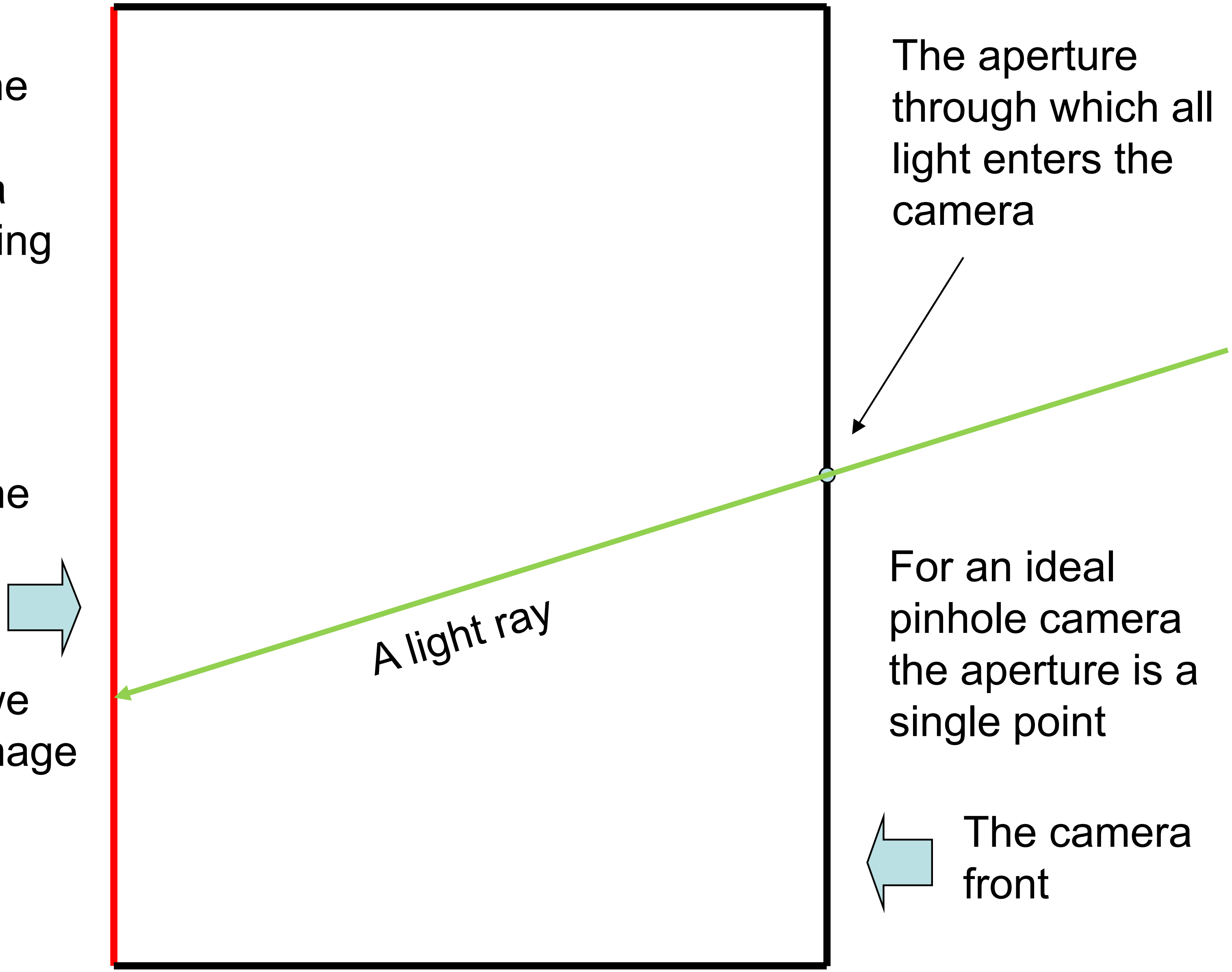


The pinhole camera model

Each point in the image plane is illuminated by a single ray passing through the aperture

The image plane

This is where we measure the image



The aperture through which all light enters the camera

For an ideal pinhole camera the aperture is a single point

The camera front

The pinhole camera model

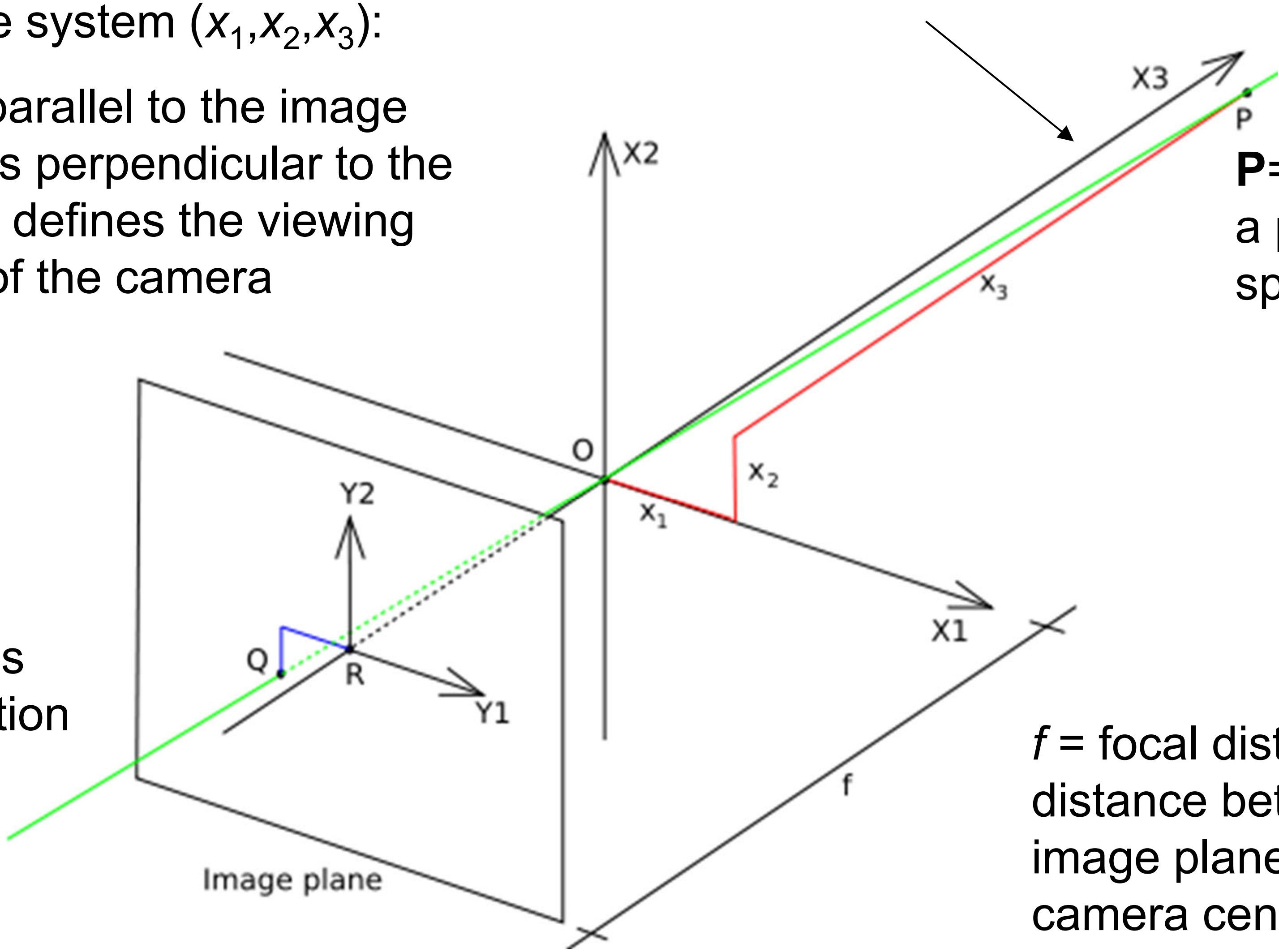
- Mathematically we need only know the location of the image plane and the aperture
  - The rest is physics + practical implementation
- In the literature, the aperture point is also called
  - camera center
  - camera focal point

The pinhole camera model

The image plane and the camera center define a camera-centered coordinate system (x_1,x_2,x_3):

x_1,x_2 are parallel to the image plane, x_3 is perpendicular to the plane and defines the viewing direction of the camera

Principal or optical axis



P=(x_1,x_2,x_3) is a point in 3D space

Q=(y_1,y_2) is the projection of **P**

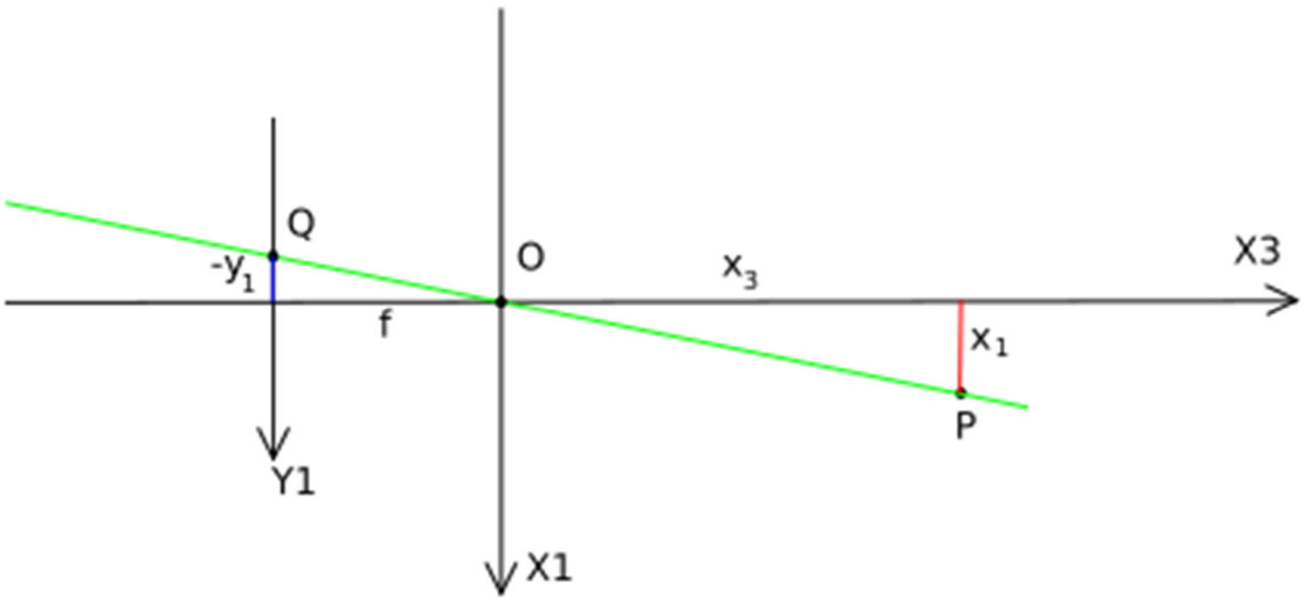
f = focal distance, the distance between the image plane and the camera center

The pinhole camera model

- **R** is the point where the optical axis intersects the image plane
  - The *principal point* or the *image center*
- The (x_1,x_2) plane is the *principal plane* or *focal plane*
- The **green line** is the *projection line* of point **P**
  - All points on the line are projected onto **Q**
  - Alternatively: the projection line of **Q**

The pinhole camera model

- If we look at the camera coordinate system along the x_2 axis:



Two similar triangles give:

$$\frac{-y_1}{f} = \frac{x_1}{x_3} \quad \text{or} \quad y_1 = -\frac{f\ x_1}{x_3}$$

The pinhole camera model

- Looking along the x_1 axis gives a similar expression for y_2
- This can be summarized as:

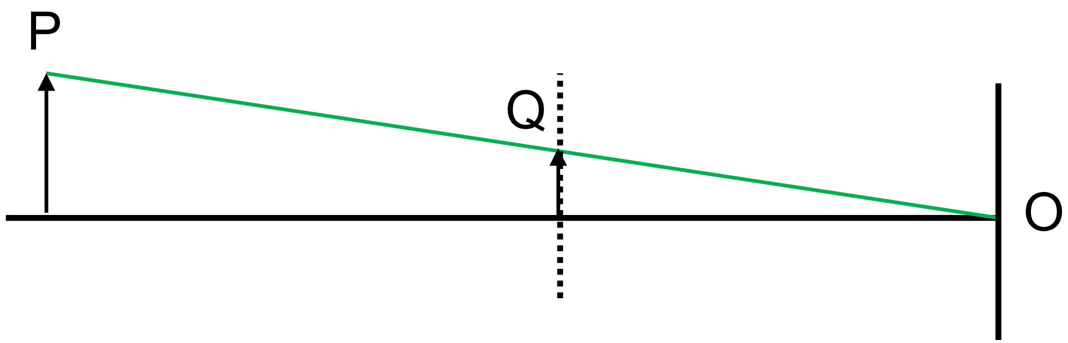
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = -\frac{f}{x_3} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The virtual image plane

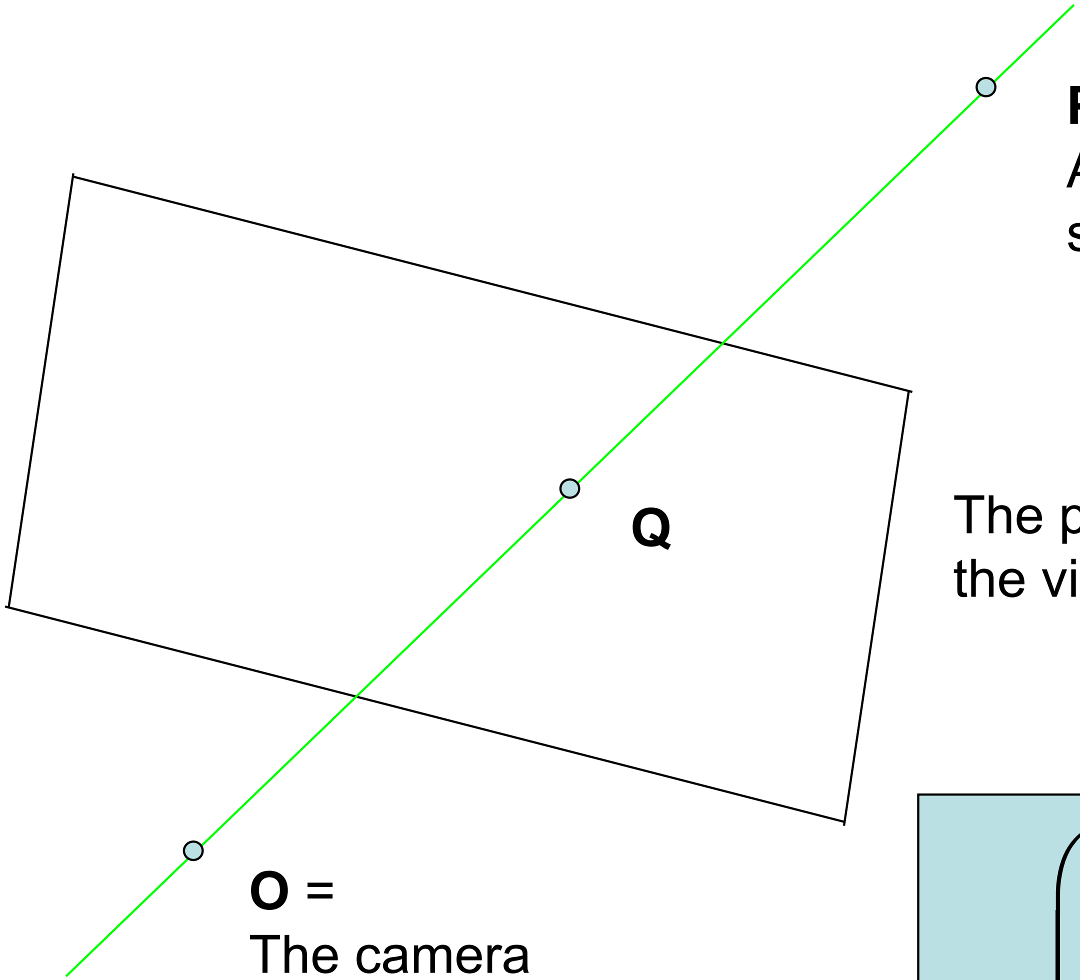
- The projected image is rotated 180^o relative to how we “see” the 3D world
  - Reflection in both y_1 and y_2 coordinates = rotation
- Must be de-rotated before we can view it
  - In the film based camera, the image is manually rotated
  - In the digital camera this is taken care of by reading out the pixels in the “rotated” order
- Mathematically this is equivalent to placing the image plane *in front* of the focal point

The virtual image plane

- Projection lines works as before: from **P** through the focal point and intersect at **Q**
- This defines the *virtual image plane*
  - Cannot be realized in practice
  - Produces the same image as the rotated image from the real image plane



The virtual image plane



P =
A point in 3D space

The projection of **P** onto the virtual image plane

O =
The camera focal point

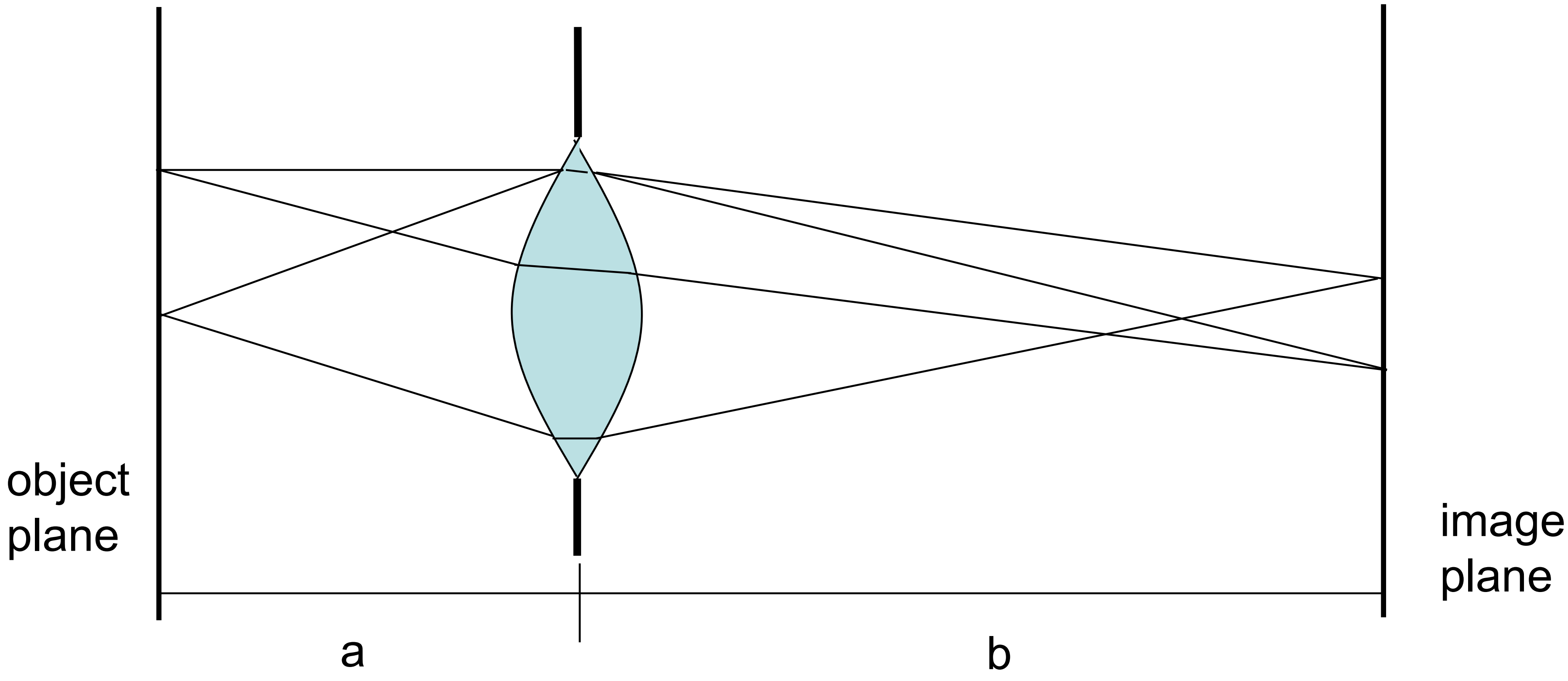
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \frac{f}{x_3} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Lenses vs. infinitesimal aperture

- The pinhole camera model doesn't work particularly well in practice since
  - If we make the aperture small, too little light enters the camera
  - If we make the aperture larger, the image becomes blurred
- Solution: we replace the aperture with a lens or a system of lenses

Thin lenses

- The simplest model of a lens
- Focuses all points in an *object plane* onto the image plane



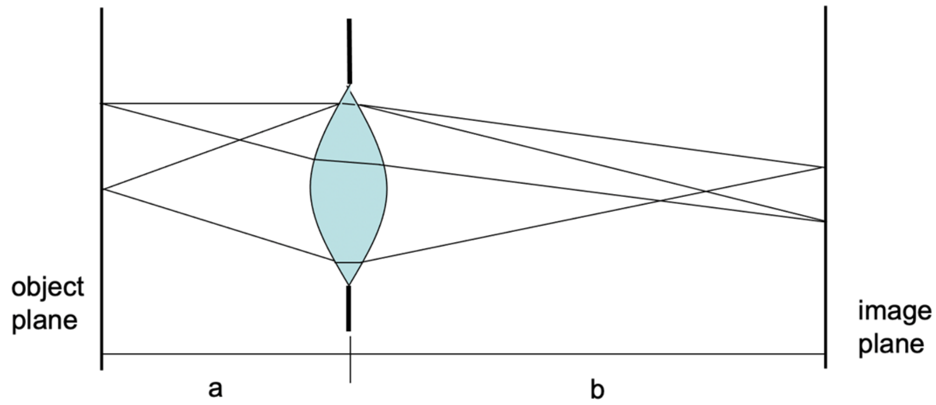
The object plane

- The object plane consists of all points that appear sharp when projected through the lens onto the image plane
- The object plane is an ideal model of where the “sharp points” are located
  - In practice: the object plane may be non-planar: e.g. described by the surface of a sphere
  - The shape of the object plane depends on the quality of the lens (or lens system)
  - For thin lenses the object plane can often be approximated as a plane

Thin lenses

- The thin lens is characterized by a single parameter: the *focal length* f_L

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f_L}$$



- To change *a* (distance to object plane), we need to change *b* since f_L is constant
- $a = \infty$ for $b = f_L$!

End of Part 2.
Questions?

Topics we will return to and deepen:

- Emission, Absorption
  - Lectures: “The IR sensor”, “3D visualization”
- Surface reflection
  - Lecture: “3D visualization”
- The plenoptic function and special light cameras
  - Lecture: “Specialized cameras”
- The pinhole camera & The virtual image plane
  - Lectures: “Camera calibration 1 & 2”
- Lenses
  - Next lecture (Image Formation, Part 3)