

Examination in

## TSBB09 Image Sensors (Bildsensorer)

*Time:* 2021-01-11

*Room:* Distance

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*Equipment permitted:* Since the exam is at a distance, you may use the course material. However, it is NOT allowed to get help from other people!

The written examination consists of 3 parts, one part for each of the three course aims in the curriculum. It comprises a total of 24 exercises and gives a maximum of 30 points. Each part consists of 6 exercises where the student should demonstrate ability to explain concepts, phenomena, etc (type A exercises), and 2 additional exercises that test a deeper understanding of various topics in the course, for example, in terms of simpler calculations (type B exercises).

Type A exercises give at most 1 point each. Type B exercises give at most 2 points each.

To pass with grade 3: At least a total of 4 points each in each of the three parts and at least 14 points in total.

To pass with grade 4: At least a total of 6 points each in each of the three parts and at least 20 points in total.

To pass with grade 5: At least a total of 8 points each in each of the three parts and at least 26 points in total.

It is enough to write your name and personal number on the first page of all your pages. Arrange the pages in exercise order.

You can write your answers either in Swedish or English.

Good luck!  
Maria Magnusson

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## PART I: STANDARD & IR IMAGE SENSORS

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### Exercise 1 (A, 1p) Image formation

Explain the meaning of the *plenoptic function*, as a function of position  $\mathbf{x} = (x_1, x_2, x_3)$  and direction  $\hat{\mathbf{n}} = (n_1, n_2, n_3)$ , where  $||\hat{\mathbf{n}}|| = 1$ .

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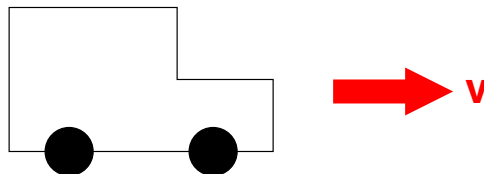
### Exercise 2 (A, 1p) Image formation



In the image above two images from the same camera have been stitched together. The two blue rectangles mark two positions where the corner from one of the images is visible in a slightly darker color. This is caused by a physical phenomena. What is the name of it? Also, describe it briefly.

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### Exercise 3 (A, 1p) Image sensors



See above. A van is moving to the right. A photo is taken with a camera with rolling shutter. Show how the photo is distorted and explain why.

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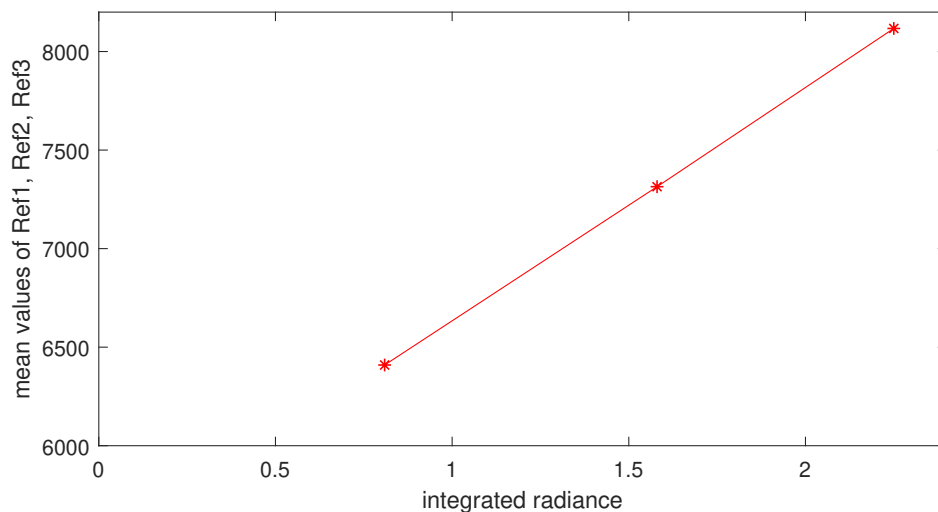
**Exercise 4 (A, 1p) Image sensors**

Bayer images which have been converted to RGB images, sometimes exhibit artifacts in the vicinity of sharp edges. Explain why these artifacts appear.

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**Exercise 5 (A, 1p) Infrared and Multispectral Imaging**

An infrared sensor can measure temperature, but needs to be calibrated. One sensor uses 3 blackbody references at three different temperatures, 3.5°, 22° and 34°. The integrated radiances for these temperatures are 0.809, 1.58, and 2.25 [W/(m<sup>2</sup>,sr)], respectively. Images of the blackbody references were taken and the mean was calculated for each image to 6409, 7314, and 8117. The integrated radiances and the three mean values are plotted against each other, see the figure below.



Why does not the line cross the origin?

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**Exercise 6 (A, 1p) Infrared and Multispectral Imaging**

An IR camera that is set to measure human skin temperature gives wrong temperature for e.g. aluminum. Why?

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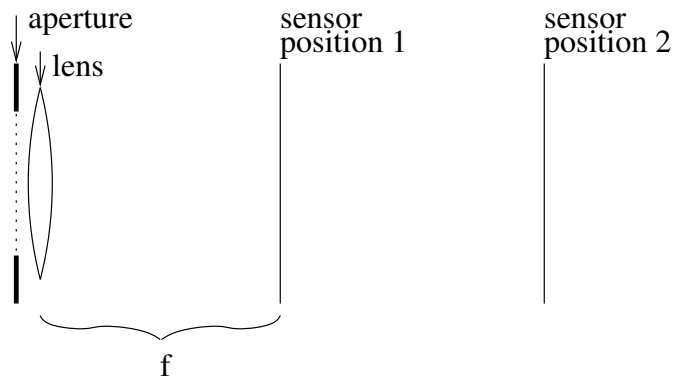
**Exercise 7 (B, 2p) Image sensors**

Regard the CCD-sensor and the CMOS-sensor.

- Explain why it is *not* possible to read only a part of the image with one of the two techniques and why it is possible with the other technique.
- Which technique can be equipped with active pixel sensors and what is the advantage with this?

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**Exercise 8 (B, 2p) Image formation**



The figure shows an aperture and a lens and two positions of the sensor plane.  $f$  denotes the focal length. Suppose that the aperture is a rectangular function  $\Pi(x)\Pi(2y)$ .

- What is the pointspread function in sensor position 1? Give an equation.
- What is the pointspread function in sensor position 2? It is enough to describe the shape with words.

*Hints:*

The Fourier transform of  $\Pi(x)$  is  $\text{sinc}(u)$ .

The scaling theorem states that if  $\mathcal{F}[f(x)] = F(u)$ , then  $\mathcal{F}[f(ax)] = F(u/a)/|a|$ .

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## PART II: GEOMETRY AND MULTIPLE VIEWS

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### Exercise 9 (A, 1p)

The implementation in OpenCV of Zhang's method includes a further development of Zhang's lens distortion model. The output vector of distortion coefficients is  $(k_1, k_2, p_1, p_2, k_3, k_4, k_5, k_6)$ . Suppose that  $p_1 = p_2 = k_4 = k_5 = k_6 = 0$ , then the model for radial distortion is

$$\begin{cases} \check{x} = x + x \cdot (k_1 f(r) + k_2 g(r) + k_3 h(r)) \\ \check{y} = y + y \cdot (k_1 f(r) + k_2 g(r) + k_3 h(r)) \end{cases} ,$$

where  $(x, y)$  are the undistorted normalized image coordinates,  $(\check{x}, \check{y})$  are the distorted normalized image coordinates and  $r = \sqrt{x^2 + y^2}$ . Give the functions  $f(r)$ ,  $g(r)$  and  $h(r)$ !

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### Exercise 10 (A, 1p)

When do tangential distortion occurs in a camera? Include the words **lens** and **sensor** in your answer.

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### Exercise 11 (A, 1p)

A camera with optical zoom is calibrated for one zoom setting, leading to the intrinsic camera matrix

$$\mathbf{A}_1 = \begin{pmatrix} 1100 & 0 & 190 \\ 0 & 900 & 134 \\ 0 & 0 & 1 \end{pmatrix} .$$

The camera zooms out a factor of 2 (so that the objects in the image become smaller). Give the new  $\mathbf{A}_2$  matrix.

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### Exercise 12 (A, 1p)

The following matrix describes a homography,

$$\mathbf{H} = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & 1 \end{pmatrix} .$$

Give another matrix that describes exactly the same homography.

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### Exercise 13 (A, 1p)

When creating a panorama in spherical coordinates, you need to know the relation between the unit sphere and the normalized image plane. Give an equation how to calculate a 3D point on the unit sphere  $(x_s, y_s, z_s)$  from a point on the normalized image plane  $(x_n, y_n, 1)$ .

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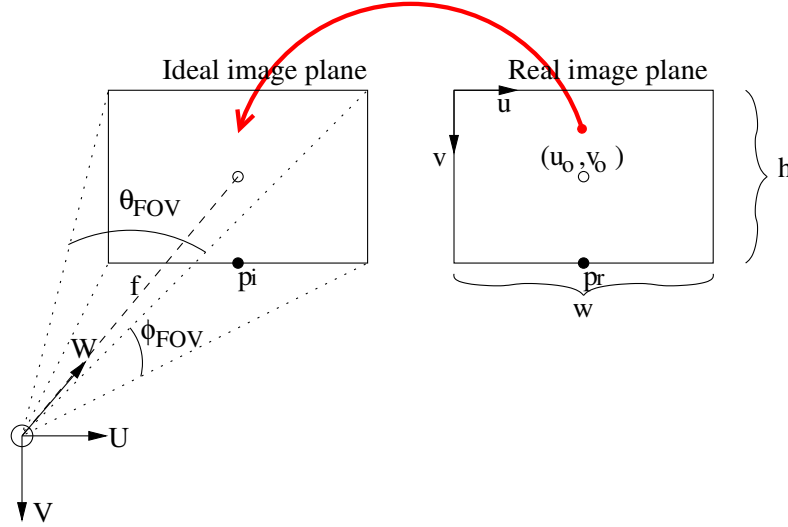
### Exercise 14 (A, 1p)

Explain carefully a method to improve the panorama stitching result in Exercise 2. With the described method the undesired darker corners will not be visible.

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**Exercise 15** (B, 2p)

When using panorama stitching in spherical coordinates, the horizontal and vertical fields of view,  $\theta_{\text{FOV}}$  and  $\phi_{\text{FOV}}$ , are determined. See the figure below, which describes the geometry inside the camera with the camera coordinate system  $(U, V, W)$  and the ideal image plane. The real image plane, which is measured in pixels, is shown to the right. It is actually positioned on top of the ideal image plane. Its width and height,  $w = 720$  and  $h = 360$ , are indicated in the figure.



The camera calibration matrix (which gives the intrinsic camera parameters) can be written as:

$$\mathbf{A} = \begin{pmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \alpha & 0 & w/2 \\ 0 & \beta & h/2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 700 & 0 & 360 \\ 0 & 700 & 180 \\ 0 & 0 & 1 \end{pmatrix}.$$

Note that we, for simplicity, have assumed  $\gamma = 0$ ,  $u_0 = w/2$ , and  $v_0 = h/2$ .

Derive an equation for the vertical field of view  $\phi_{\text{FOV}}$  in terms of a simple expression containing  $\alpha$ ,  $\beta$ ,  $w$  and/or  $h$ . Also calculate a numerical value for  $\phi_{\text{FOV}}$ .

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**Exercise 16** (B, 2p)

Using Zhang's method, we can determine the intrinsic camera parameters in terms of the matrix

$$\mathbf{A} = \begin{pmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In this method, several views of a calibration plane are shown to the camera. The method uses that

- a homography is determined by  $k_1$  parameters,
  - a 3D rotation matrix is determined by  $k_2$  angles and
  - a 3D translation vector is determined by  $k_3$  coordinates.
- a) Give the minimal values of  $k_1$ ,  $k_2$  and  $k_3$  and use this to motivate the minimum number of distinct views of the calibration plane to determine  $\mathbf{A}$ .
- b) Same question as in a), but suppose that  $\gamma = 0$ .

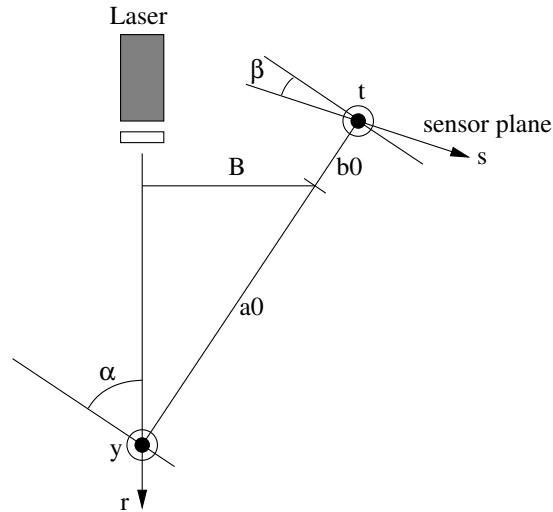
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## PART III: NON-STANDARD IMAGE SENSORS

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### Exercise 17 (A, 1p) Range cameras

A common setup for a sheet-of-light range camera is shown in the figure above. Which angle  $\beta$  should the sensor be tilted to get perfect sharpness over the entire sensor plane? Assume that  $f = 17$  mm,  $b_0 = 17.5$  mm,  $\alpha = 60^\circ$ .



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### Exercise 18 (A, 1p) Range cameras

Range measurements can be made based on *time-of-flight* techniques or on *laser triangulation*. Compare the two techniques in terms of occlusion.

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### Exercise 19 (A, 1p) 3D visualization

The program below calculates a normal projection image of a 3D-volume along the y-direction,  $P(x, z)$ . Modify the program so that it calculates a MIP image,  $M(x, z)$ , instead.

```
for z=-127 to 128
  for x=-127 to 128
    P(x,z):=0;
    y:=-127;
    do
      P(x,z):=P(x,z)+f(x,y,z);
      y:=y+1;
    while (y<129)
  end;
end;
```

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**Exercise 20 (A, 1p) Specialized cameras**

A pinhole camera and a push-broom camera sample the plenoptic function in different ways. Describe this difference!

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**Exercise 21 (A, 1p) Specialized cameras**

The technique with “coded aperture” can be used to extend the depth-of-field of a camera. The equation  $|f_k * x - y|^2$  is of importance. From an unsharp image  $y$ , a sharp image  $x$ , can be calculated. Describe how  $f_k$  looks like and how the shape of  $f_k$  varies for different depths  $k$ .

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**Exercise 22 (A, 1p) Specialized cameras**

The flexible depth of field camera moves the sensor during exposure. What is the advantage with this camera?

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**Exercise 23 (B, 2p) 3D visualization**

The following equation can be used for 3D-visualization of a diffuse object in gray scale:

$$\mathbf{I}_{diffuse} = \mathbf{I} \cos \phi$$

- a) Give an equation for 3D-visualization of a gold object. Suppose white light. All the following numbers (slightly rounded off) from the 3D visualization lab must be included in the equation:

	ambient			diffuse			specular			shininess
gold	0.25	0.20	0.07	0.75	0.61	0.23	0.63	0.56	0.37	14

- b) Do the same as in a), but suppose yellow light.

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**Exercise 24 (B, 2p) Range images**

Multipath interference (MPI) is a problem that occurs on some time-of-flight (ToF) sensors.

- a) Why does this occur?
- b) For each of these sensors, state whether it is sensitive to MPI or not:
- Velodyne Lidar
  - Ouster OS-1 Lidar
  - Kinect v2 ToF Camera